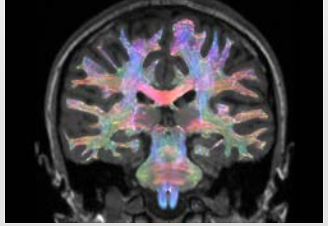


ANS Virtual Winter Meeting

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Sensitivity Analysis of the 1-D Thermal Stratification Model via Forward and Adjoint Sensitivity Methods


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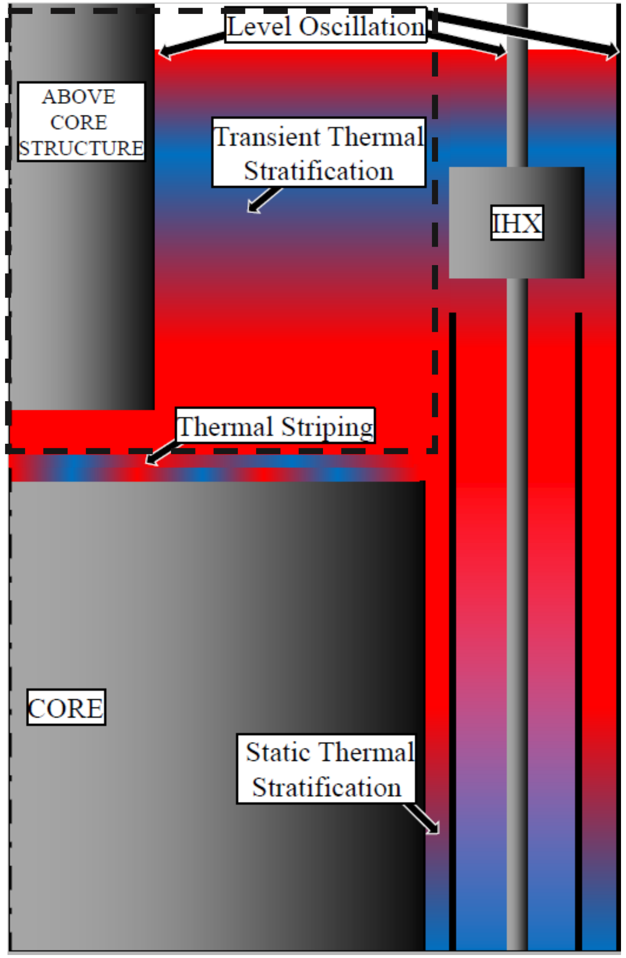
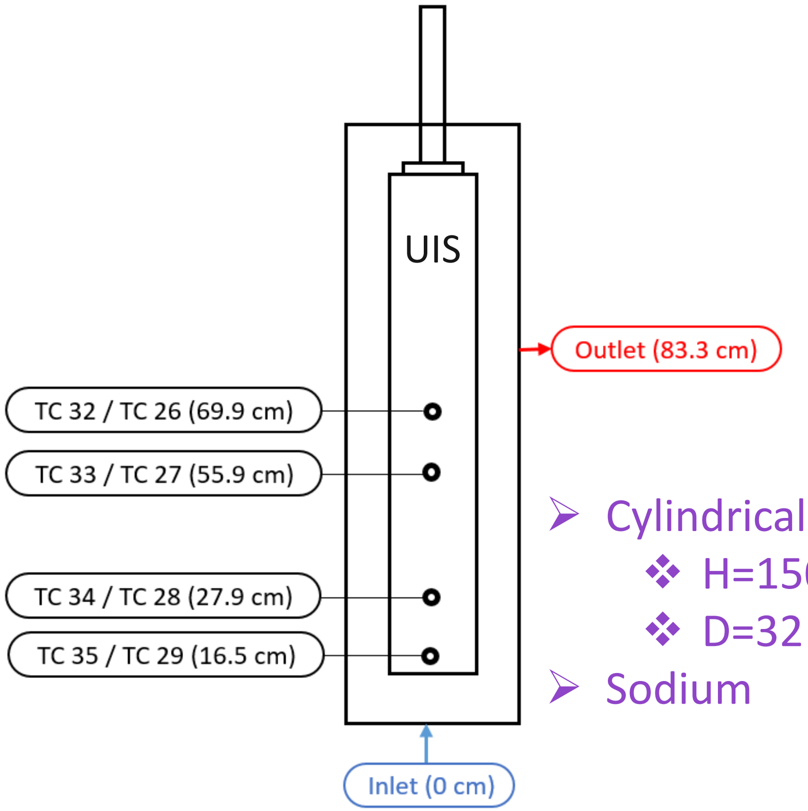
Background – Thermal stratification in nuclear systems

- Thermal stratification
  - Formation of stratified layers of coolant with a large temperature gradient
- Nuclear systems involved
  - High-Temperature Gas-Cooled Reactors (HTGR)
  - Small-Modular Boiling-Water Reactors (SMBWR)
  - **Pool-type Sodium-Cooled Fast Reactors (SFR)**
  - ...
- Concerns
  - Leads to neutronic and thermal-hydraulic instabilities
  - Causes thermal fatigue crack growth
  - **Impedes natural circulation**



Background – Development of the 1-D TS model (1)

The Thermal Stratification Testing Facility (TSTF)



- Cylindrical
  - H=150 cm
  - D=32 cm
- Sodium

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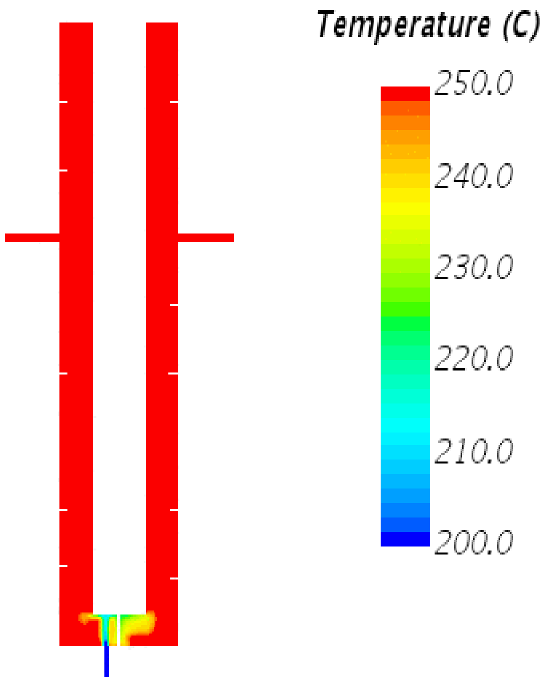
CFD modeling and simulation



Massachusetts Institute of Technology

1-D TS model

$$\rho_{amb} c_{p,amb} \frac{\partial T_{amb}}{\partial t} + \rho_{amb} c_{p,amb} \bar{u}_z \frac{\partial T_{amb}}{\partial z} - \frac{\partial}{\partial z}\left(k_{amb} \frac{\partial T_{amb}}{\partial z}\right) = \frac{N_{jet}}{A_{amb}} c_{p,jet} \rho_{jet} Q'_{jet} (T_{jet} - T_{amb})$$

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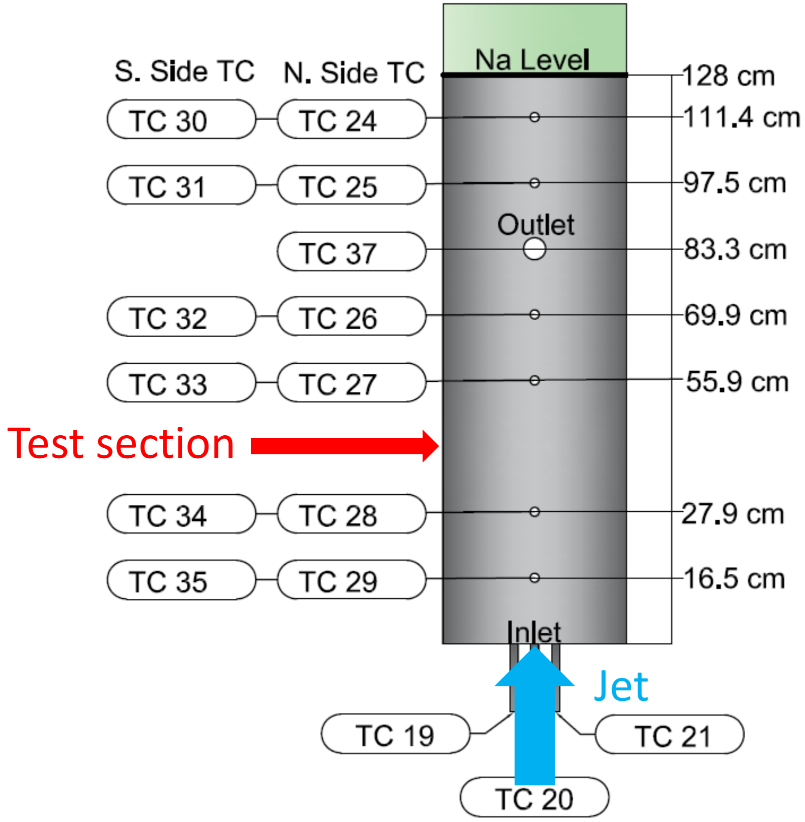
Background – Development of the 1-D TS model (2)

Jet volumetric flow rate

$$\rho_{amb} c_{p,amb} \frac{\partial T_{amb}}{\partial t} + \rho_{amb} c_{p,amb} \bar{u}_z \frac{\partial T_{amb}}{\partial z} - \frac{\partial}{\partial z}\left(k_{amb} \frac{\partial T_{amb}}{\partial z}\right) = \frac{N_{jet}}{A_{amb}} c_{p,jet} \rho_{jet} Q'_{jet} (T_{jet} - T_{amb})$$

Heat capacity of the ambient fluid　Thermal conductivity of the ambient fluid　Jet temperature

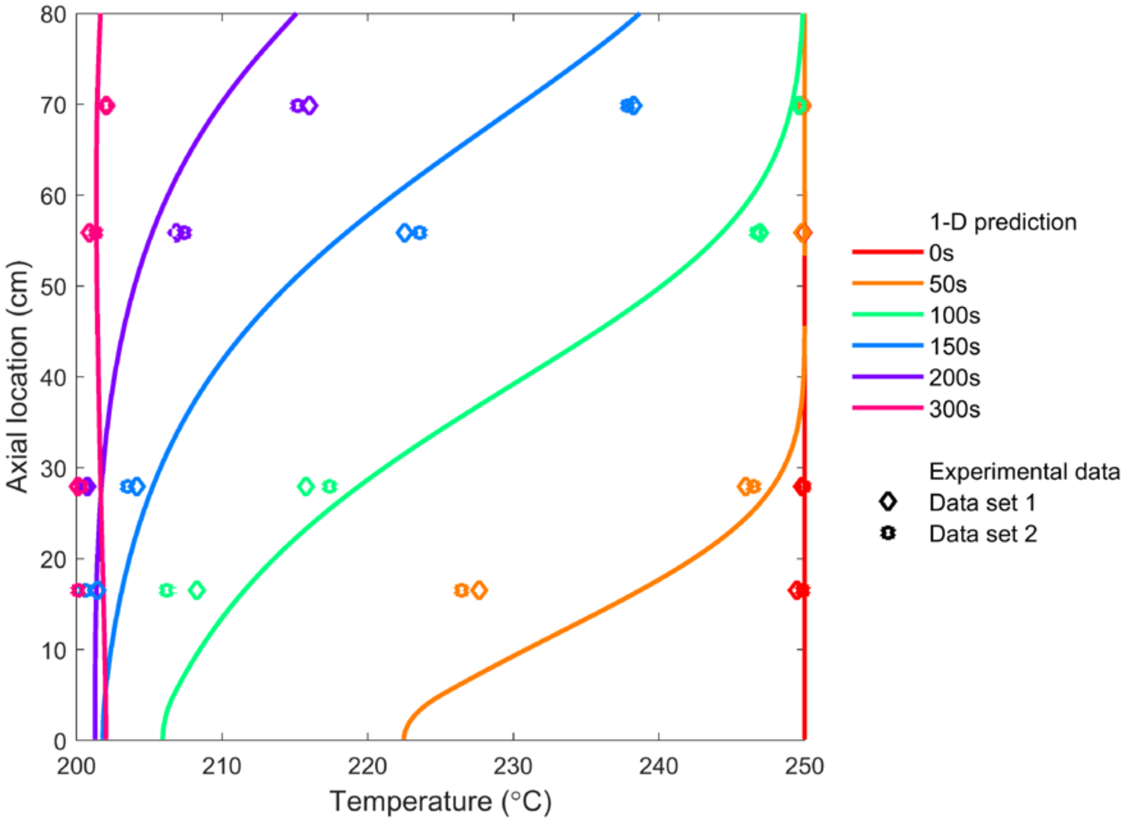


Test conditions

$T_{jet} = 200°C$

$T_{amb} = 250°C$

$Q_{jet} = 0.38\ L/s$



The Thermal Stratification Testing Facility (TSTF)

Objective of this work

To investigate the sensitivity of the temperature gradient of the ambient fluid in the test section, which serves as a good quantitative metric (figure of merit - FOM) reflecting the severity of the thermal stratification phenomenon, with respect to four parameters.

Parameters considered

- Jet volumetric flow rate Q_{jet}
- Jet temperature T_{jet}
- Heat capacity of the ambient fluid $C_{p,amb}$
- Thermal conductivity of the ambient fluid $k_{c,amb}$

Method employed

- Conventional forward sensitivity method
- Advanced adjoint sensitivity method

Outline

- ❖Sensitivity analysis with conventional forward sensitivity method
- ❖Sensitivity analysis with adjoint sensitivity method
- ❖Additional findings from the sensitivity analysis
- ❖Uncertainty quantification by using the adjoint sensitivities
- ❖Summary and conclusions

Forward sensitivity method

1-D thermal stratification model

⇩

Temperature distribution $\boldsymbol{T}$

⇩

Temperature gradient distribution $\boldsymbol{J}$

⇩

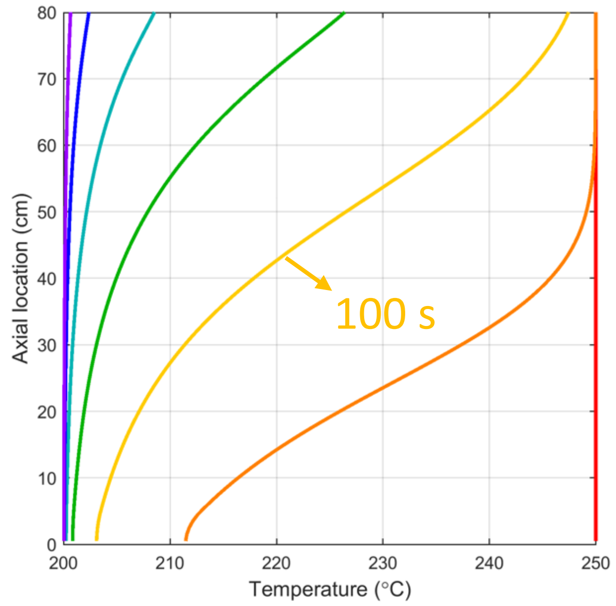
Relative sensitivity $\boldsymbol{S_r}$

$$S_{r,\theta} = \frac{\delta J}{\delta \theta}\frac{\theta_0}{J_0} = \frac{J\big(T(\theta_0 + \Delta\theta)\big) - J\big(T(\theta_0)\big)}{\Delta\theta}\frac{\theta_0}{J_0}$$

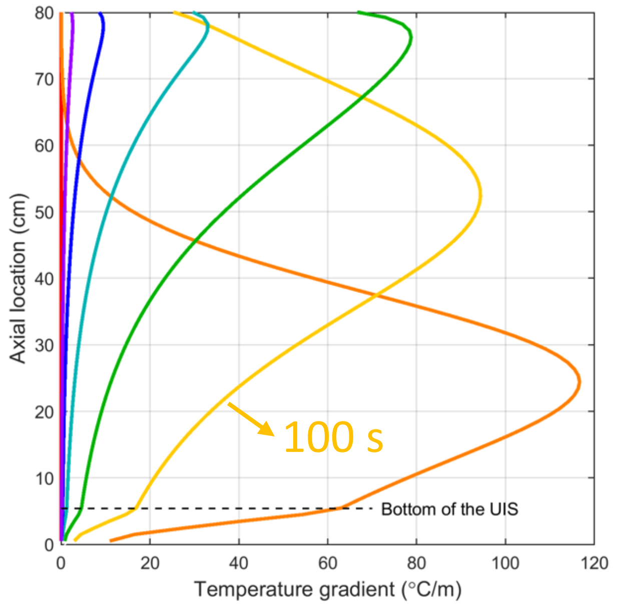
where $\theta = Q_{jet}, T_{jet}, C_{p,amb}$, or $k_{c,amb}$

Minimal computational expense

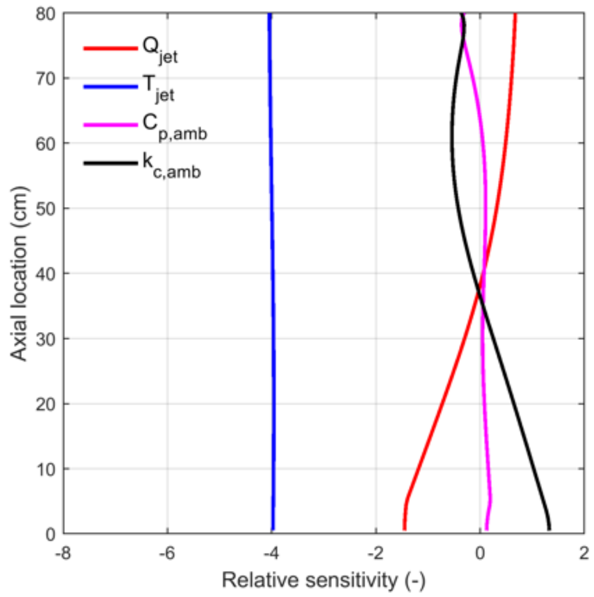
- Nominal condition × 1
- Variation introduced × 4
- 5 in total



Temperature distribution $\boldsymbol{T}$



Temperature gradient distribution $\boldsymbol{J}$



Relative sensitivity $\boldsymbol{S_r}$

Can we avoid repetitively solving the 1-D system?

Adjoint sensitivity method - theory

$\delta \boldsymbol{J}$: A variation in temperature gradient
$\delta \boldsymbol{T}$: A variation in temperature
$\delta \boldsymbol{\theta}$: A variation in the input parameters

- $\delta J(T) = \frac{dJ(T)}{dT} \delta T$
  - Repetitively solving 1-D TS model for different $\delta \boldsymbol{T}$
  - Cast 1-D TS model into the residual form: $\boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta}) = \boldsymbol{0}$
  - $\delta \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta}) = \boldsymbol{0} = \frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{T}} \delta \boldsymbol{T} + \frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \delta \boldsymbol{\theta}$
  - $\delta \boldsymbol{J}(\boldsymbol{T}) = \frac{d\boldsymbol{J}(\boldsymbol{T})}{d\boldsymbol{T}} \delta \boldsymbol{T} + \boldsymbol{\Phi}^T \cdot \boldsymbol{0} = \frac{d\boldsymbol{J}(\boldsymbol{T})}{d\boldsymbol{T}} \delta \boldsymbol{T} + \boldsymbol{\Phi}^T \left(\frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{T}} \delta \boldsymbol{T} + \frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \delta \boldsymbol{\theta} \right)$
  - $\delta \boldsymbol{J}(\boldsymbol{T}) = \left(\frac{d\boldsymbol{J}(\boldsymbol{T})}{d\boldsymbol{T}} + \boldsymbol{\Phi}^T \frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{T}} \right) \delta \boldsymbol{T} + \boldsymbol{\Phi}^T \frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \delta \boldsymbol{\theta}$
- $\delta \boldsymbol{J}(\boldsymbol{T}) = \boldsymbol{\Phi}^T \frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \delta \boldsymbol{\theta}$ (This is true when $\frac{d\boldsymbol{J}(\boldsymbol{T})}{d\boldsymbol{T}} + \boldsymbol{\Phi}^T \frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{T}} = \boldsymbol{0} \rightarrow$ the adjoint equation)
- $\boldsymbol{\Phi}^T = -\frac{d\boldsymbol{J}(\boldsymbol{T})}{d\boldsymbol{T}} \cdot \left(\frac{\partial \boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta})}{\partial \boldsymbol{T}} \right)^{-1}$

Adjoint sensitivity method - application

The space-time discretized form of 1-D thermal stratification model:

- $\rho_{amb} c_{p,amb} \frac{\partial T_{amb}}{\partial t} + \rho_{amb} c_{p,amb} \bar{u}_z \frac{\partial T_{amb}}{\partial z} - \frac{\partial}{\partial z}\left(k_{amb} \frac{\partial T_{amb}}{\partial z}\right) = \frac{N_{jet}}{A_{amb}} c_{p,jet} \rho_{jet} Q'_{jet} (T_{jet} - T_{amb})$
- $\rho_n^{m-1} c_{p,n}^{m-1} \frac{T_n^m - T_n^{m-1}}{\Delta t} + \rho_n^{m-1} c_{p,n}^{m-1} u_{z,n} \frac{T_n^m - T_{n-1}^m}{\Delta z} - \frac{2}{\Delta z} k_n^{m-1} \left(\frac{T_{n+1}^m - T_n^m}{2\Delta z} - \frac{T_n^m - T_{n-1}^m}{2\Delta z}\right) = \frac{1}{A_{amb,n}} c_{p,jet} \rho_{jet} Q'_{jet,n} (T_{jet} - T_n^{m-1})$
- $F_n^m = A_{n,n-1}^m T_{n-1}^m + A_{n,n}^m T_n^m + A_{n,n+1}^m T_{n+1}^m - B_n^m T_n^{m-1} - C_n^m = 0$
- $\boldsymbol{F}(\boldsymbol{T}, \boldsymbol{\theta}) = \boldsymbol{0}$ with $(M+1) \times N$ equations $(M+1)$ time steps and N spatial steps (300 time steps and 83 spatial steps)
- Expressions of $\frac{\partial \boldsymbol{F}(\boldsymbol{T},\boldsymbol{\theta})}{\partial \boldsymbol{T}}$, $\frac{\partial \boldsymbol{F}(\boldsymbol{T},\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$, and $\frac{d\boldsymbol{J}(\boldsymbol{T})}{d\boldsymbol{T}}$
- Calculation of $\boldsymbol{\Phi}^T = -\frac{d\boldsymbol{J}(\boldsymbol{T})}{d\boldsymbol{T}} \cdot \left(\frac{\partial \boldsymbol{F}(\boldsymbol{T},\boldsymbol{\theta})}{\partial \boldsymbol{T}}\right)^{-1}$
- Calculation of $\delta \boldsymbol{J}(\boldsymbol{T}) = \boldsymbol{\Phi}^T \frac{\partial \boldsymbol{F}(\boldsymbol{T},\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \delta \boldsymbol{\theta}$
- Absolute sensitivity $\boldsymbol{S}^m = (\boldsymbol{\Phi}^m)^T \frac{\partial \boldsymbol{F}(\boldsymbol{T},\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ at time step m

Relative sensitivity

$$\boldsymbol{S}_{r}^{m}{}_{\in(N,4)} = \begin{pmatrix} S_{1,Q}^m \cdot \frac{Q_{jet}}{J_1^m} & S_{1,T}^m \cdot \frac{T_{jet}}{J_1^m} & S_{1,C_p}^m \cdot \frac{C_p(T_1^m)}{J_1^m} & S_{1,k_c}^m \cdot \frac{k(T_1^m)}{J_1^m} \\ \vdots & \vdots & \vdots & \vdots \\ S_{N,Q}^m \cdot \frac{Q_{jet}}{J_N^m} & S_{N,T}^m \cdot \frac{T_{jet}}{J_N^m} & S_{N,C_p}^m \cdot \frac{C_p(T_N^m)}{J_N^m} & S_{N,k_c}^m \cdot \frac{k(T_N^m)}{J_N^m} \end{pmatrix}_{\in(N,4)}$$

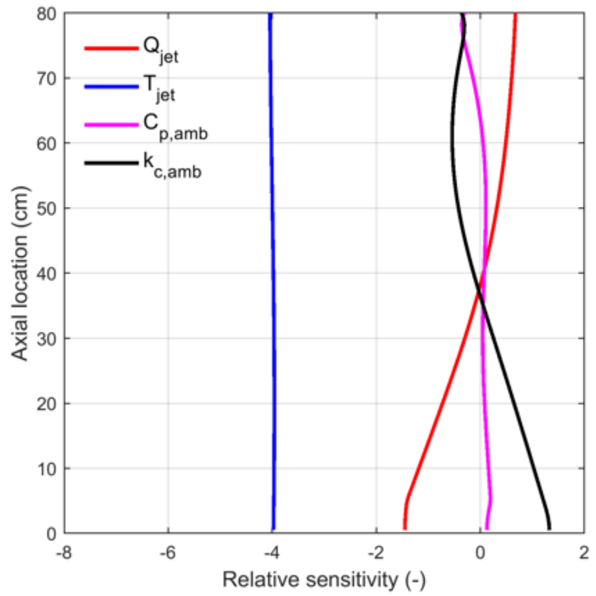
Semi-relative sensitivity

$$\boldsymbol{S}_{sr}^{m}{}_{\in(N,4)} = \begin{pmatrix} S_{1,Q}^m \cdot Q_{jet} & S_{1,T}^m \cdot T_{jet} & S_{1,C_p}^m \cdot C_p(T_1^m) & S_{1,k_c}^m \cdot k(T_1^m) \\ \vdots & \vdots & \vdots & \vdots \\ S_{N,Q}^m \cdot Q_{jet} & S_{N,T}^m \cdot T_{jet} & S_{N,C_p}^m \cdot C_p(T_N^m) & S_{N,k_c}^m \cdot k(T_N^m) \end{pmatrix}_{\in(N,4)}$$

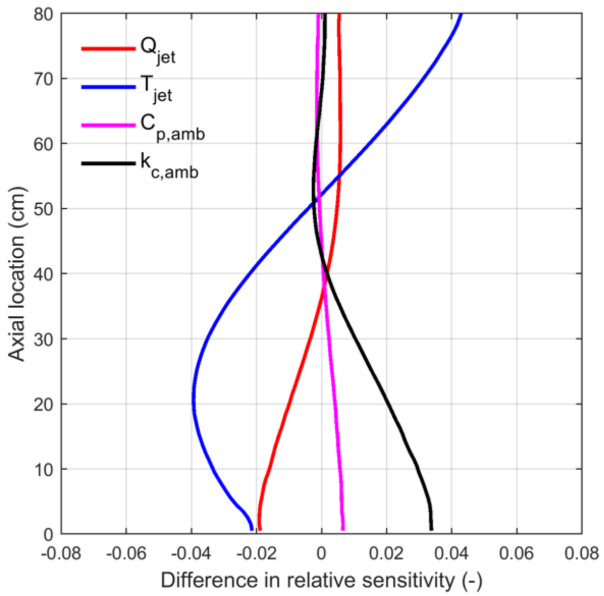
C. Lu and Z. Wu, 2020. "Sensitivity Analysis of the 1-D SFR Thermal Stratification Model via Discrete Adjoint Sensitivity Method," *Nuclear Engineering and Design*, **370**.
https://doi.org/10.1016/j.nucengdes.2020.110920

Are we confident about the application process?

Adjoint sensitivity method - verification



S_r from forward method



Difference in S_r

Minimal computational expense

- Nominal condition × 1
- Computation of matrices × 1
  - ✓ $\frac{\partial F(T,\theta)}{\partial \theta}$
  - ✓ $\frac{dJ(T)}{dT}$
  - ✓ $\frac{\partial F(T,\theta)}{\partial T}$
  - ✓ Φ^T
- 2 in total (compared to 5 in total for forward method)

Parameter investigated	Max. abs. difference in S_r	Forward method	Adjoint method
Q_{jet}	0.02	-1.45	-1.47
T_{jet}	0.04	-3.96	-4.00
$C_{p,amb}$	0.01	0.13	0.14
$k_{c,amb}$	0.03	1.33	1.36

The adjoint method is more efficient when N_{output} is small and N_{input} is large (Wang, 2013).

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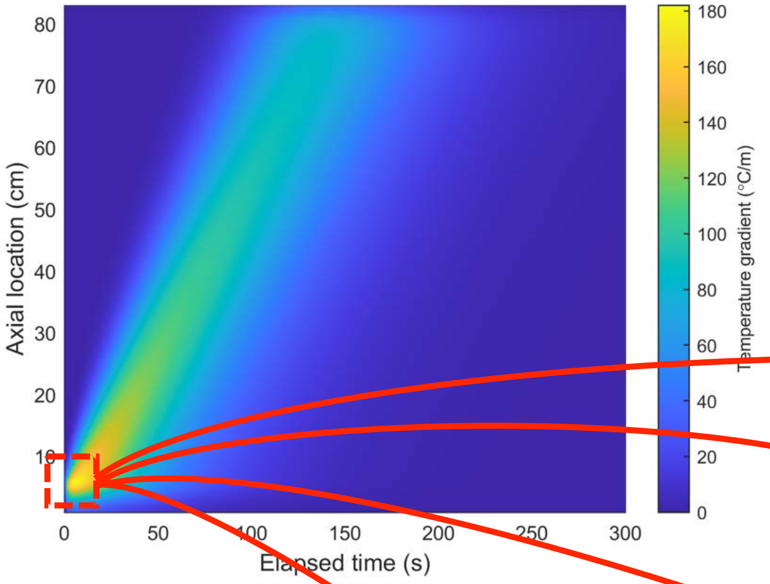
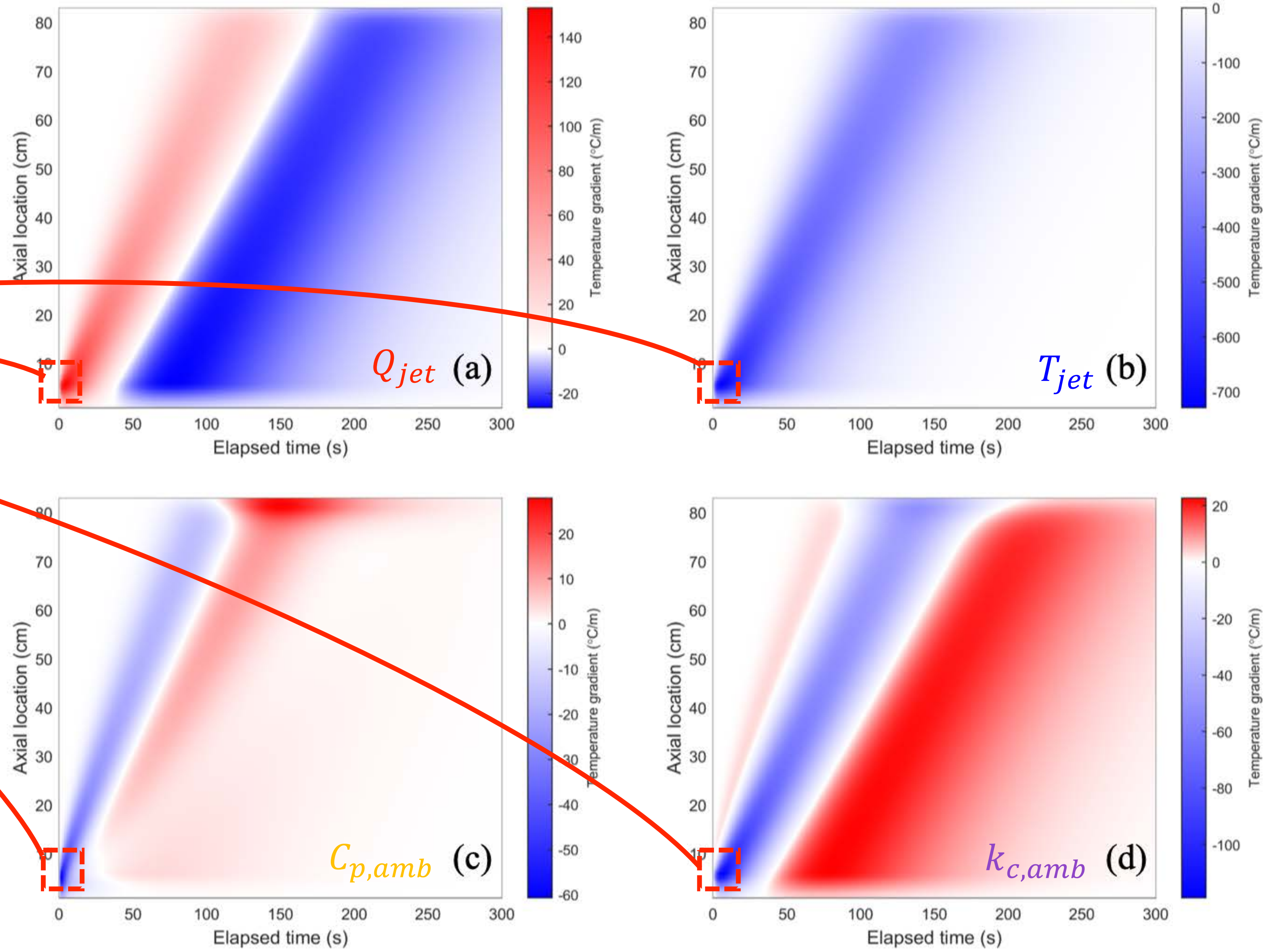
Additional findings from the adjoint sensitivity analysis





Change of temperature gradient caused by

- Q_{jet}, $C_{p,amb}$, and $k_{c,amb}$
  - Positive or negative
- T_{jet}
  - Always negative

Sensitivity of Max temperature gradient

- 150 °C/m for Q_{jet}
- -730 °C/m for T_{jet}
- -30 °C/m for $C_{p,amb}$
- -120 °C/m for $k_{c,amb}$

What else to do with these sensitivity information?

Uncertainties quantification

- **Deterministic method with adjoint sensitivities**

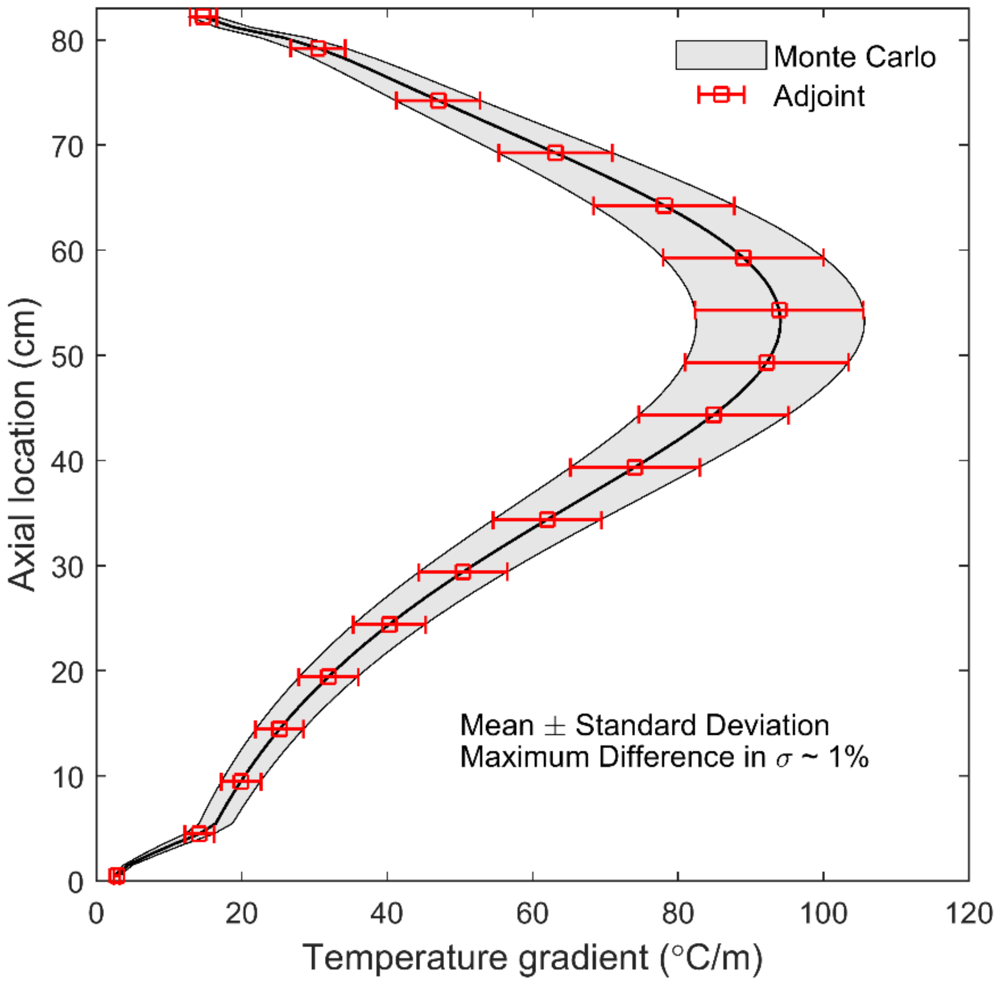
uncertainty of a response — uncertainty of the m^{th} input parameter

$$\sigma_R^2 = \sum_m (s_m \sigma_m)^2$$

adjoint sensitivity of the response to the m^{th} input parameter

- **Verification of the correctness**
  - Uncertainty of predicted temperature gradient at 100s elapsed time
  - Q_{jet} (±3%), T_{jet} (±3%), $C_{p,amb}$ (±3%), $k_{c,amb}$ (±5%)
  - Monte Carlo method with 500,000 realizations

- **Comparison of the computational cost**
  - Adjoint sensitivity method → equivalent to 2 times
  - Monte Carlo method → at least 20,000 times



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Summary and conclusions

- **Performed a sensitivity study of the temperature gradient**
  - Conventional forward method ⇔ Advanced discrete adjoint method
  - Mutual verification of the correctness of both methods
  - Advanced discrete adjoint method is $\sim 50\%$ computational costly
- **Investigated four parameters**
  - Q_{jet} sensitivity (+/-), max 150 °C/m
  - T_{jet} sensitivity (-), max -730 °C/m
  - $C_{p,amb}$ sensitivity (+/-), max -30 °C/m
  - $k_{c,amb}$ sensitivity (+/-), max -120 °C/m
- **Performed an uncertainty quantification of the temperature gradient**
  - Deterministic method with adjoint sensitivities ⇔ Monte Carlo method
  - Mutual verification of the correctness of both methods
  - Deterministic method with adjoint sensitivities is $\sim 0.01\%$ computational costly

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