Advances in Inverse Transport Methods and Applications to Neutron Tomography

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Back to the Future: Nuclear Energy for Global Sustainability

Outline of This Talk

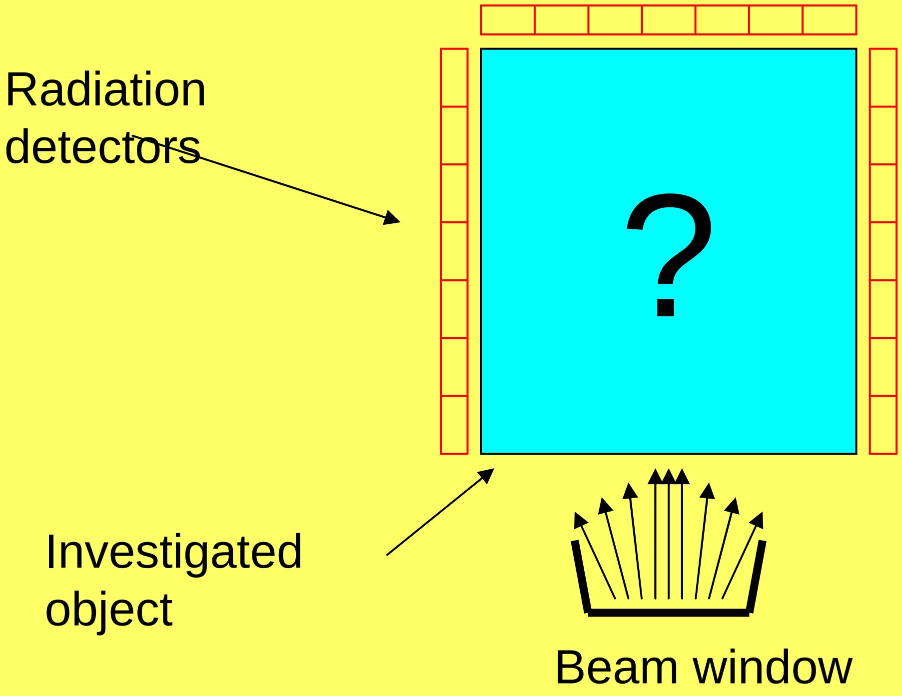
- Introduction
- Analytic Tomography Method
- Gradient-based Deterministic Optimization
- Stochastic-based Combinatorial Optimization
- Results
- Summary

Introduction

Solve the Inverse Problem

The purpose of our project is to infer material distribution inside an object based upon detection and analysis of radiation emerging from the object.

We are particularly interested in problems in which radiation can undergo significant scattering within the object.



Tomographic Reconstruction Methods

- Analytic methods

  - Line integral (Radon transform),

  - Fourier slice theorem (FST)

  - Back projection reconstruction (BPR)

- Iteration methods

  forward model & inverse update scheme

  Iteration image reconstruction (IIR) methods mainly differ in their choice of forward model and how the spatial distributions of the optical properties of the medium are updated.

Analytic Tomography Method

Model Problem

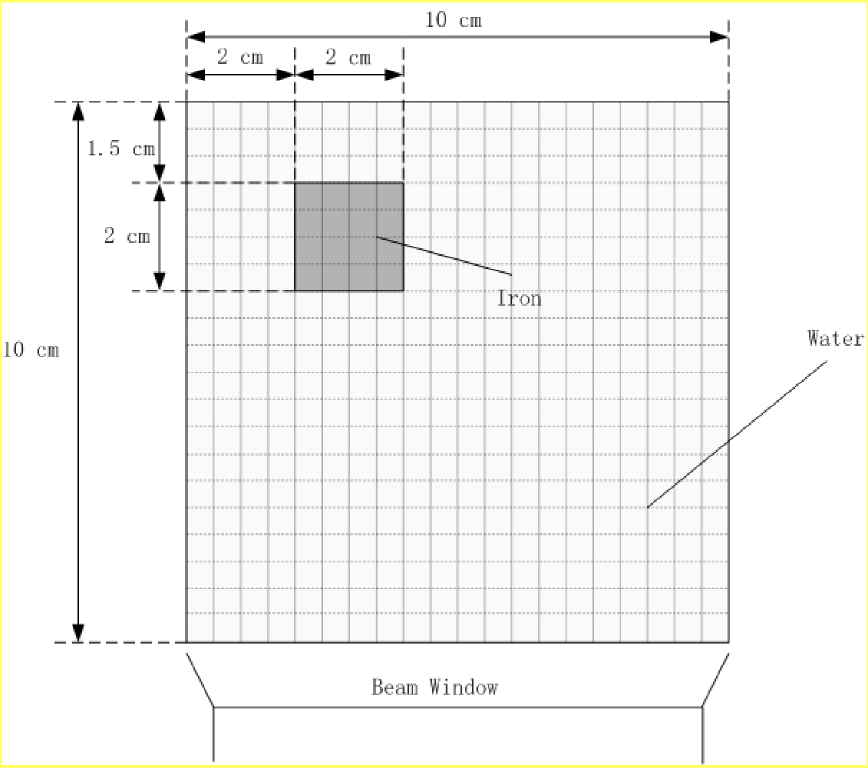


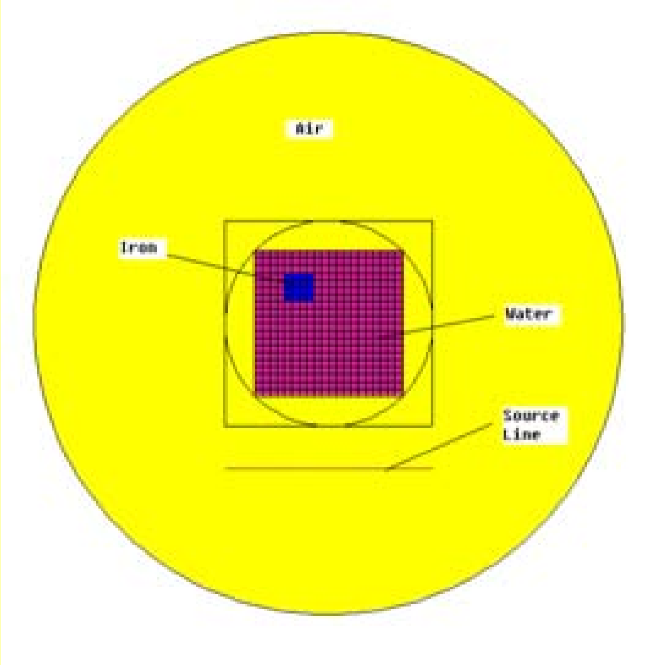
Fig: Schematic diagram of the one iron inclusion model problem (X-Y view).

Table: Physics properties of the materials in the model problem

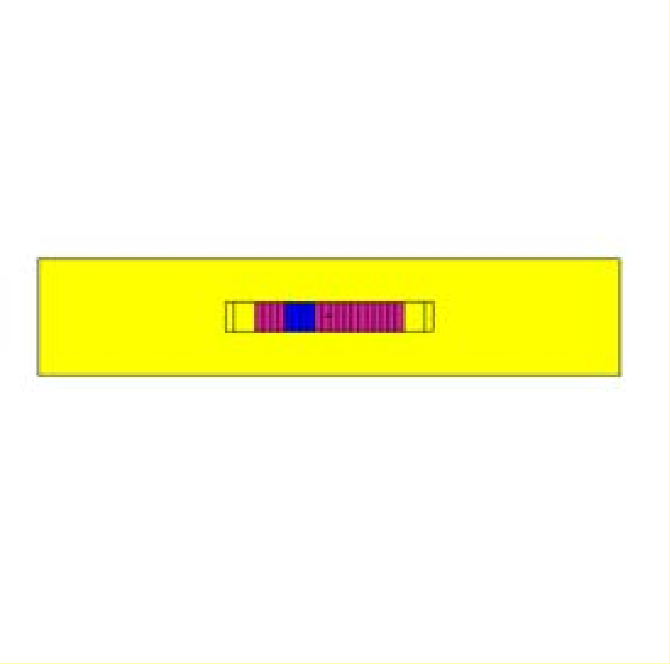
Material	Water	Iron
Σ_s (1/cm)	0.736	0.967
Σ_t (1/cm)	0.744	1.19
g (2/3A)	3.70E-2	1.20E-2
Σ_{tr} (1/cm)	0.716	1.167
mfp (cm)	1.35	0.848
c	0.990	0.820

$$\Sigma_{tr} = \Sigma_t - g\Sigma_s, \quad mfp = \frac{1}{\Sigma_t}, \quad c = \frac{\Sigma_s}{\Sigma_t}$$

MCNP[1] Modeling to Provide Projections



(a) X-Y View

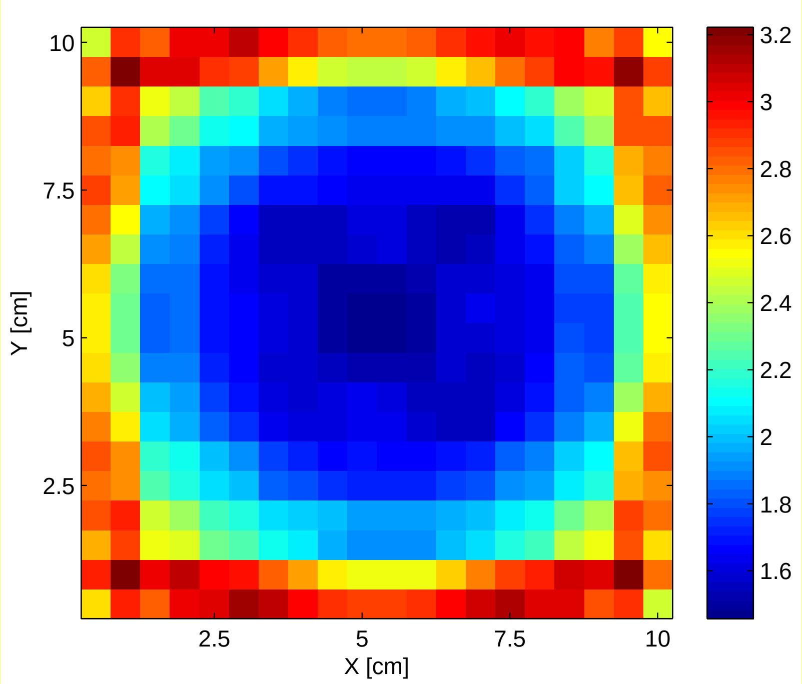


(b) X-Z View

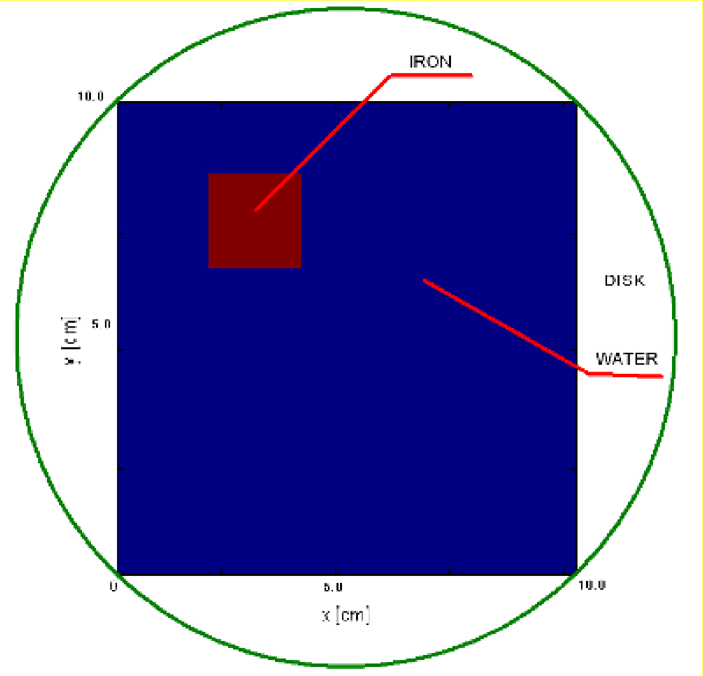
Fig: Test problem layout and experimental configuration

[1]J. Briesmeister ed., MCNP-A general Monte Carlo n-particle transport code, v-5, LA-UR-03-1987, 2003

Failure of Analytic Tomography Method to Reconstruct Total Cross Section Image for Model Problem



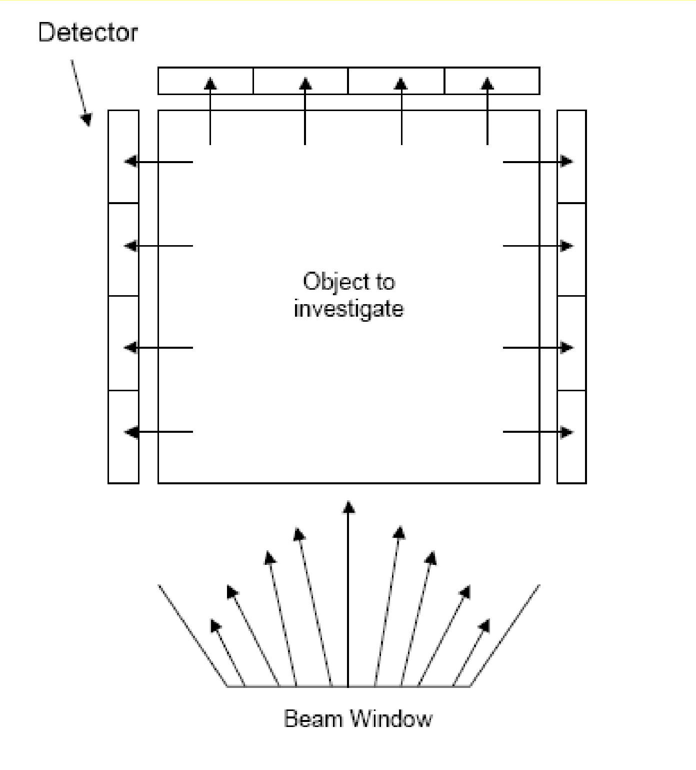
Total cross section reconstructed with FBP method for the test problem.



Geometry and material configuration of the test problem .

Gradient-based Deterministic Optimization Method

Minimization Based Optimization Approaches[2,3]



A schematic diagram of the beam-object-detector system

- Objective function

$$\Phi = \frac{1}{2}\sum_{i=1}^{N}\left(P_i - M_i\right)^2$$

- Here 'M' is experimental measurements provided by MCNP simulation
- 'P' is predicted measurements provided by forward model calculation, which is treated as a function of properties of the unknown object

$$P = P\left(\sigma_t, \sigma_s, \sigma_{s1}, \cdots\right)$$

$$\Rightarrow \Phi = \Phi\left(\sigma_t, \sigma_s, \sigma_{s1}, \cdots\right)$$

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[2]V. Scipolo, Scattered neutron tomography based on a neutron transport problem, Master Thesis, Texas A&M, 2004

[3]A. D. Klose et al., Optical tomography using the time-Independent equation of radiative transfer - part 1: forward model J. Quant. Spectrosc. Radiat. Transf. , 72, 691-713, 2002

Deterministic Optimization Results for Model Problem

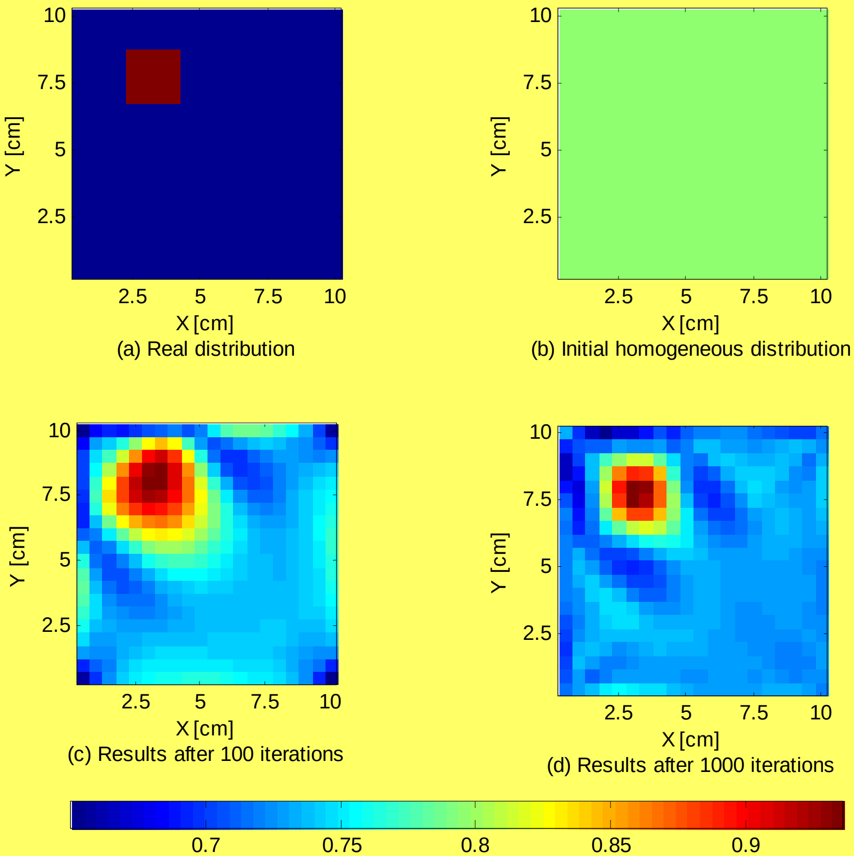
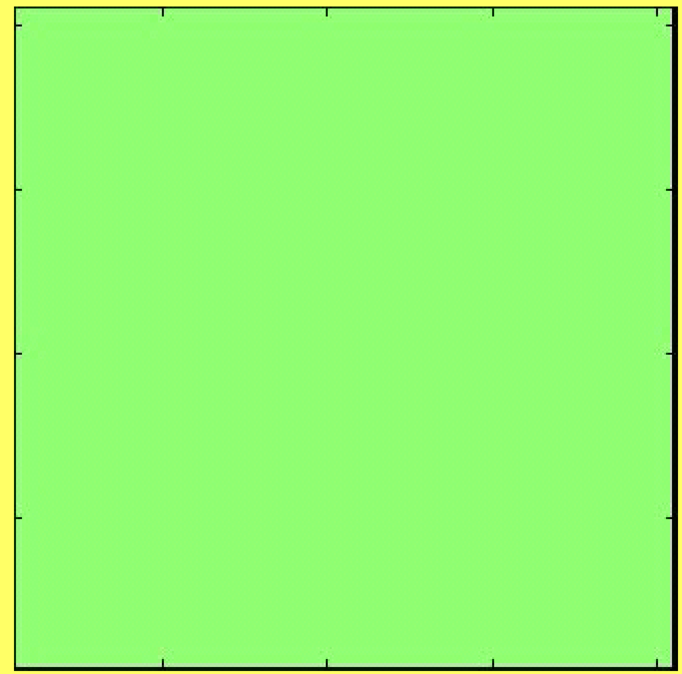


Fig: Transport cross section (Σ_{tr}) distribution obtained from deterministic CG based iterative search scheme for the one iron problem. (a) The real Σ_{tr} (background is water and square inclusion is iron). (b) Initial guess for Σ_{tr} . (c) and (d) are results after 100 and 1000 iterations, respectively.



Animation show of the transport cross section (Σ_{tr}) distribution within the object changes after each updating iteration.

Stochastic-based Combinatorial Optimization Method

Advances Devised in This Stage

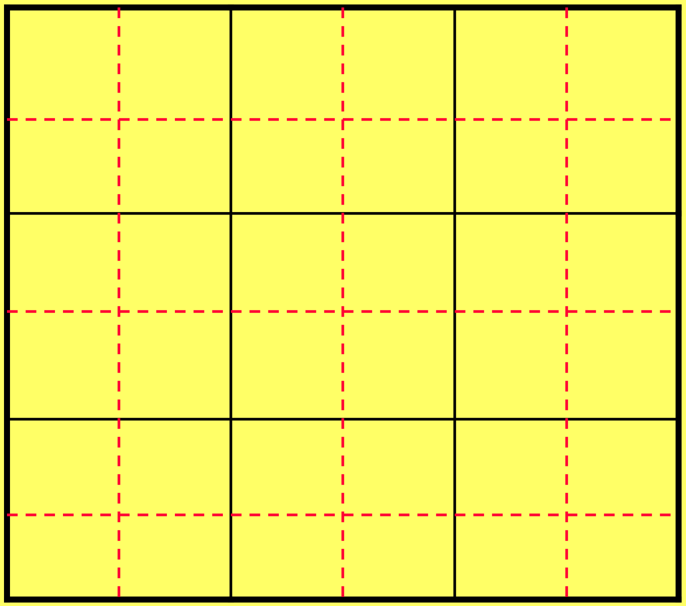
- Treat unknowns as materials instead of cross sections
- Change continuous search to discrete search
- Combine deterministic and stochastic optimization method together
- Hierarchical Approach

  - Mesh refinement

  - High fidelity forward calculation
- Search dimension reduction techniques

  - Cell grouping

  - Material restriction
- Efficient stochastic-based combinatorial optimization scheme



Material Candidate Library (MCL)

We use MCNP to generate energy collapsed one group cross sections for different materials. These information will be used to construct a material candidate library (MCL) which works as search space for the discrete search.

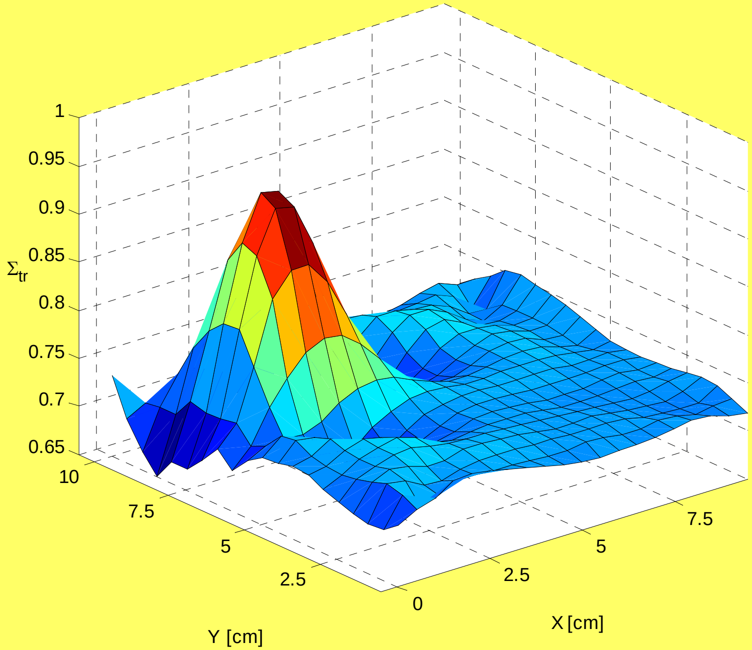
$$\overline{\sigma_i} = \frac{\int_0^{E_{th}} \sigma_i(E)\phi(E)dE}{\int_0^{E_{th}} \phi(E)dE} = \frac{F4(Fm4)\text{ Tally}}{F4\text{ Tally}}$$

$$\overline{\Sigma_i} = n\overline{\sigma_i} = \rho\frac{N_A}{M}\overline{\sigma_t}$$

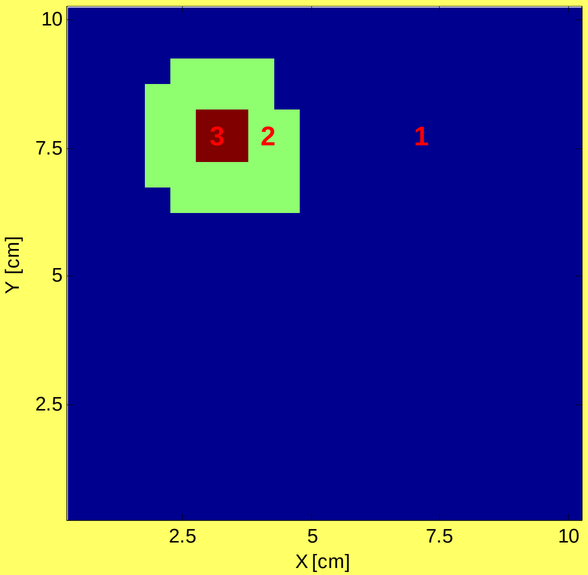
Current materials in MCL

#	Material	Σ_t (1/cm)	Σ_s (1/cm)	g ($\frac{2}{3A}$)
1	**Paraffin**	0.567	0.567	5.56E-2
2	**B-10**	5.54E+2	3.20E-1	6.67E-2
3	**Water**	0.746	0.736	3.70E-2
4	**Si**	0.110	0.102	2.37E-2
5	**Fe**	1.18	0.967	1.19E-2
6	**Nitrogen**	6.50E-4	5.50E-4	4.76E-2
7	**Cadmium (Cd)**	0.301	0.247	5.90E-3
8	**Aluminum (Al)**	9.68E-2	8.31E-2	2.47E-2
9	**Natural Uranium**	0.821	0.921	1.40E-3
10	**HEU**	2.70E+1	5.25E+1	3.61E-5

Cell Grouping Result for Model Problem

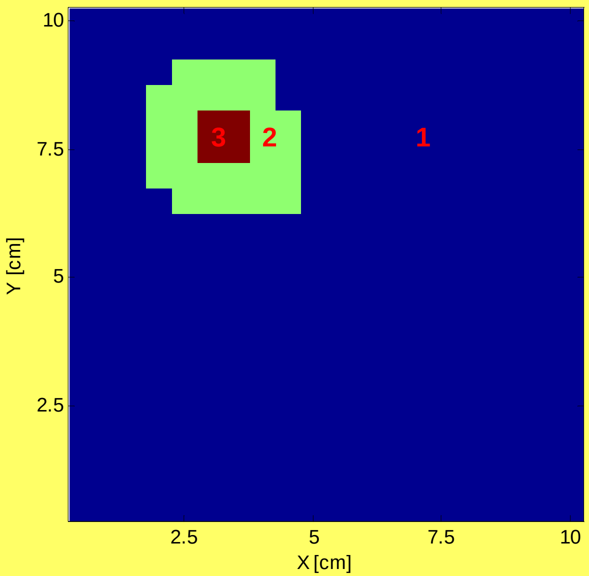


Surface plot of the Σ_{tr} distribution yielded from deterministic optimization stage



Different regions identified by the cell grouping process. (Color in this figure denotes region only, not any particular numerical value.)

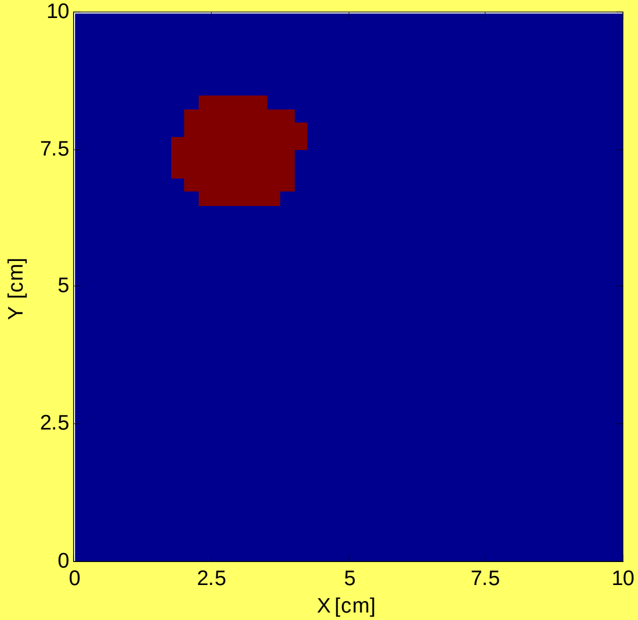
Material Restriction Result for Model Problem



Cell grouping result

- Our algorithm determined that the background material must be water; thus region 1 was always chosen to be water.
- Our algorithm also determined that the inclusion (region 3) could be any one of four different materials: iron, water, paraffin, natural uranium.

Optimization Result for Model Problem



Material distribution from the combinatorial optimization (CO) after 50 iterations. (Color in this figure denotes material only, not any particular numerical value.)

Quantitative results of the inclusion location and area with comparison to the real problem

Parameter	Actual	Opt. Result
Material	Iron	Iron
X-center (cm)	3.00	2.97
Y-center (cm)	7.50	7.49
Area (cm^2)	4.00	4.00
Iron/Water	4.17E-2	4.17E-2

Another Model Problem

Model Problem II

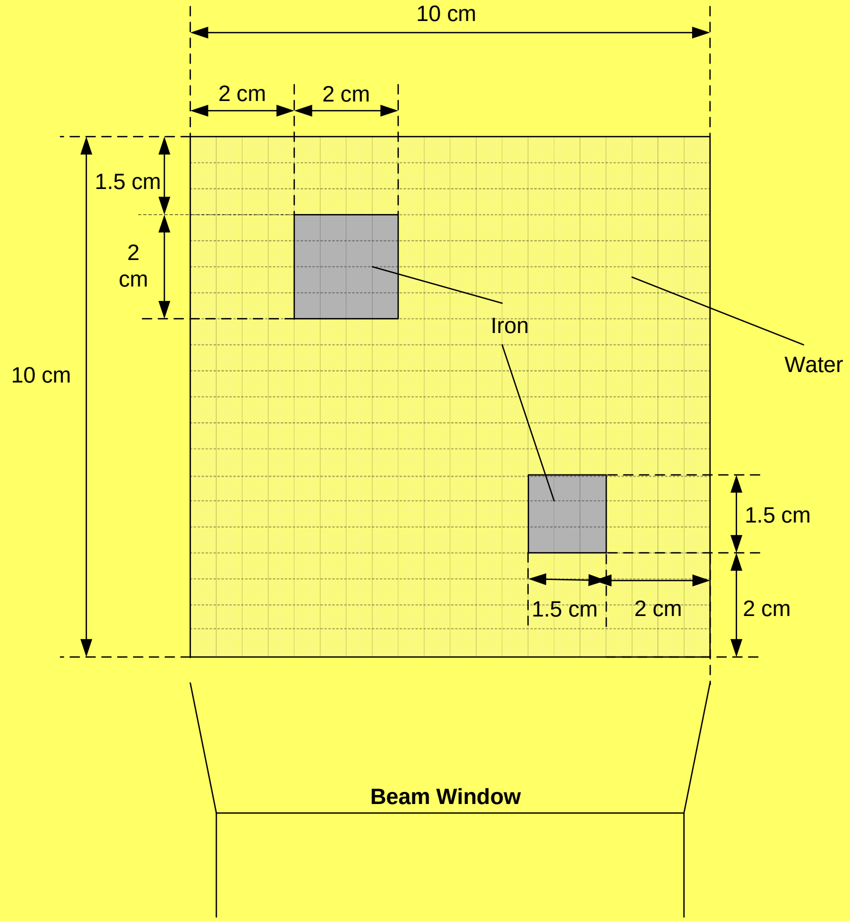


Fig: Schematic diagram of the two irons inclusion model problem (X-Y view).

Results for Model Problem II (Deterministic Stage)

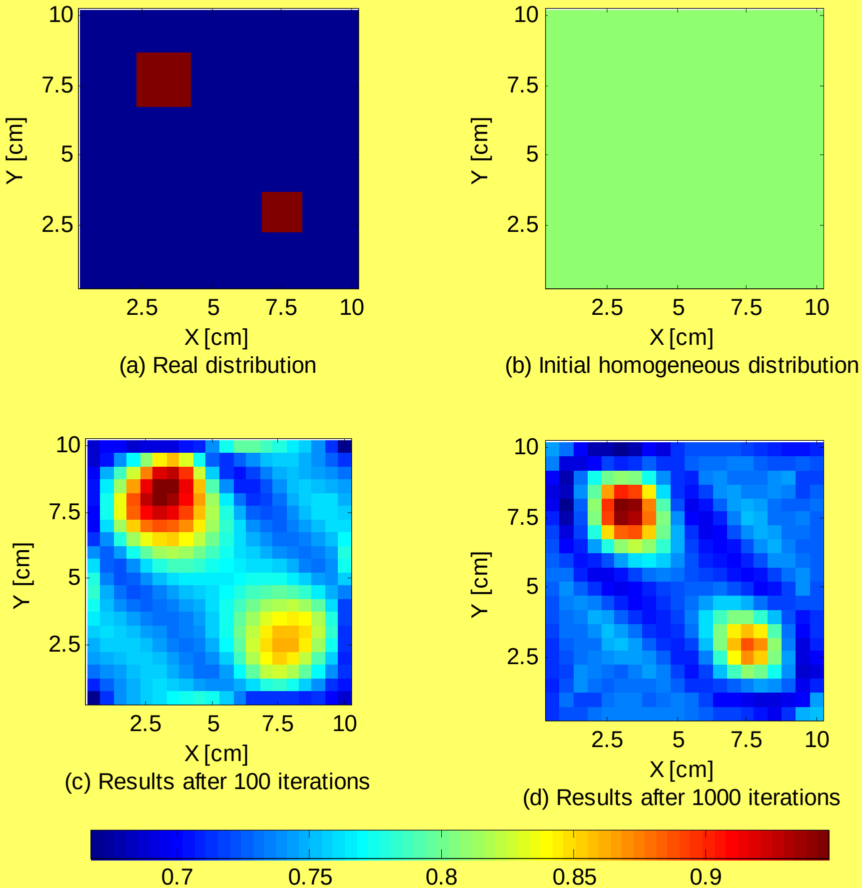


Fig. 1: Transport cross section (Σ_{tr}) distribution obtained from deterministic CG based iterative search scheme for the two irons problem. (a) The real Σ_{tr} (background is water and square inclusions are irons). (b) Initial guess for Σ_{tr} . (c) and (d) are results after 100 and 1000 iterations, respectively.

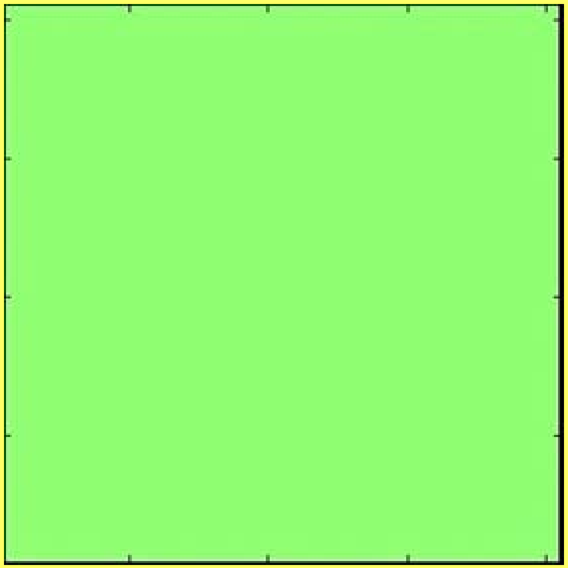
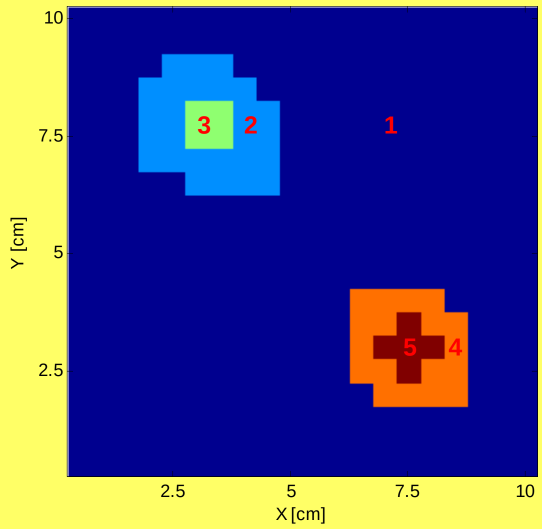
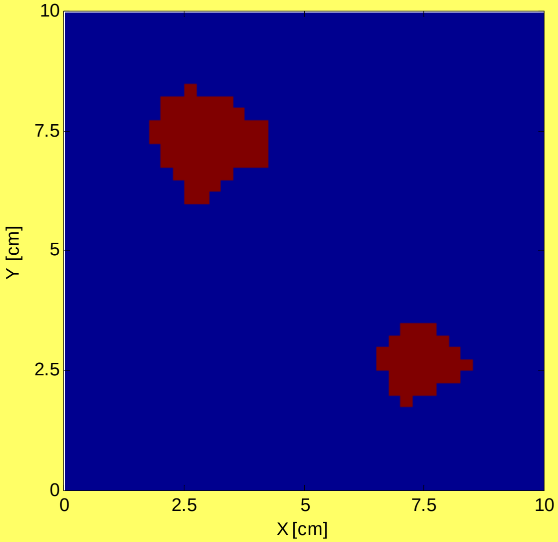


Fig. 2: Animation show of the reconstruction procedure for transport cross section (Σ_{tr}) distribution within the object changes with each iteration.

Results for Model Problem II (Stochastic Stage)



Different regions identified by the cell grouping process. (Color in this figure denotes region only, not any particular numerical value.)



Material distribution from the combinatorial optimization after 100 iterations. (Color in this figure denotes material only, not any particular numerical value.)

Quantitative result for Inclusion locations and areas with comparison to real problem

Parameter		Material	X-center (cm)	Y-center (cm)	Area (cm^2)
Inclusion 1	**Actual**	Iron	3.00	7.50	4.00
	Opt. Result	Iron	2.95	7.37	4.25
Inclusion 2	**Actual**	Iron	7.25	2.75	2.25
	Opt. Result	Iron	7.29	2.66	2.19

Summary of This Talk

- We have introduced some advances in inverse transport methods and applied them to solve 2D neutron tomography problems
- We employ multiple steps that work together to dramatically reduce the difficulty of the combinatorial optimization
- We implement a series of novel dimension-reduction techniques and integrate them to achieve the desired result
- Results from simple model problems illustrate the potential power of these ideas and feasibility of the method
- Optically thick problems with a high scattering ratio, in which analytic tomography method generally fails, are solved almost exactly with very modest computational effort

Questions?

Thank You!