PHYSOR 2018: Reactor Physics Paving the Way Towards More Efficient Systems
Cancun, Mexico, April 22-26, 2018

A MODIFIED FORM OF THE SAAF TRANSPORT EQUATION WITH FULLY VOID-COMPATIBLE FEATURE

Zeyun Wu and Samaneh Rakhshan Pouri

Department of Mechanical and Nuclear Engineering
Virginia Commonwealth University, Richmond, VA 23219 USA

zwu@vcu.edu and rakhshanpous@vcu.edu

ABSTRACT

This paper proposed a modified form of the Self Adjoint Angular Flux (SAAF) equation with the objective to make the resulting 2nd order transport equation fully compatible to problems with void regions, which remains as an inherent flaw for the conventional SAAF equation. The modified SAAF equation can be solved using source iteration and requires only the solution of an independent set of 2nd self-adjoint and conservative equation for each direction during each source iteration. The validity of the modified SAAF equation are preliminary verified with two one-dimensional one-group transport test problems with fixed sources, in which the modified SAAF equation is numerically solved using the S_N method in conjunction with a linear-continuous finite element method (CFEM) in space. The results of CFEM in both test problems show an accuracy above 90% with comparison to reference solutions. These results indicate the modified SAAF equation is an alternative candidate of the 2nd transport equation to deal with void problems.

KEYWORDS: Neutron Transport Equation, SAAF, Finite Element Method

1. INTRODUCTION

The steady-state first-order monoenergetic neutron transport equation with an isotropic scattering and an isotropic external source can be written as follows

$$\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})+\sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega})=\frac{1}{4\pi}\sigma_s(\underline{r})\phi(\underline{r})+\frac{1}{4\pi}S(\underline{r}), \tag{1}$$

where σ stands for macroscopic cross-section rather than microscopic cross-section and the rest of the notations are standard in reactor physics realm. For simple notations, we can represent the source terms as a single term

$$Q(\underline{r})=\frac{1}{4\pi}\sigma_s(\underline{r})\phi(\underline{r})+\frac{1}{4\pi}S(\underline{r}), \tag{2}$$

such that Eq.(1) becomes

$$\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})+\sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega})=Q(\underline{r}) \tag{3}$$

To derive the Self-Adjoint Angular Flux (SAAF) transport equation, one typically write Eq.(3) into the following form

$$\psi(\underline{r},\underline{\Omega}) = \frac{1}{\sigma_t(\underline{r})}\left[Q(\underline{r}) - \underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})\right], \tag{4}$$

and substitutes it into the angular flux streaming term [i.e., $\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})$] in Eq.(3) to yield (with a little manipulation)

$$-\underline{\Omega}\cdot\underline{\nabla}\frac{1}{\sigma_t(\underline{r})}\left[\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})\right] + \sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega}) = Q(\underline{r}) - \underline{\Omega}\cdot\underline{\nabla}\frac{Q(\underline{r})}{\sigma_t(\underline{r})}\ , \tag{5}$$

By using the following identity

$$\underline{\Omega}\cdot\underline{\nabla}\left[\frac{1}{\sigma_t(\underline{r})}\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})\right] = \underline{\nabla}\cdot\left[\frac{1}{\sigma_t(\underline{r})}\underline{\underline{\Omega\Omega}}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})\right] = \underline{\nabla}\cdot\frac{\underline{\underline{\Omega\Omega}}}{\sigma_t(\underline{r})}\underline{\nabla}\psi(\underline{r},\underline{\Omega}), \tag{6}$$

we can rewrite Eq.(5) as the follows

$$-\underline{\nabla}\cdot\frac{\underline{\underline{\Omega\Omega}}}{\sigma_t(\underline{r})}\underline{\nabla}\psi(\underline{r},\underline{\Omega}) + \sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega}) = Q(\underline{r}) - \underline{\Omega}\cdot\underline{\nabla}\frac{Q(\underline{r})}{\sigma_t(\underline{r})}. \tag{7}$$

Eq.(7) is usually referred to as the standard SAAF equation [1], which has a long standing issue when applying to problems with void regions because the inverse total cross section term appears in the equation [2-4]. In this paper we propose a modified form for the SAAF equation that avoids the appearance of the inverse total transport cross section term with the purpose to make the SAAF equation universally applicable to problems with void regions.

2. A MODIFIED FORM OF THE SAAF TRANSPORT EQUATION

To obtain a SAAF equation without the inverse total cross section term, we manipulate Eq.(1) in a different way. Instead of representing Eq.(1) as the form of Eq.(4), we write it as the following form

$$\sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega}) = Q(\underline{r}) - \underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega}). \tag{8}$$

Meanwhile, we manipulate Eq.(1) a little bit to make the $\sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega})$ term appear in the streaming term as a whole in the transport equation by multiplying $\sigma_t(\underline{r})$ at both sides of the equation

$$\sigma_t(\underline{r})\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega}) + \sigma_t^2(\underline{r})\psi(\underline{r},\underline{\Omega}) = \sigma_t(\underline{r})Q(\underline{r}). \tag{9}$$

With the notice of the following identity

$$\underline{\Omega}\cdot\underline{\nabla}\left[\sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega})\right] = \sigma_t(\underline{r})\left[\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})\right] + \psi(\underline{r},\underline{\Omega})\left[\underline{\Omega}\cdot\underline{\nabla}\sigma_t(\underline{r})\right]. \tag{10}$$

Eq.(9) can be written as the following form

$$\underline{\Omega}\cdot\underline{\nabla}\left[\sigma_t(\underline{r})\psi(\underline{r},\underline{\Omega})\right] + \left[\sigma_t^2(\underline{r}) - \underline{\Omega}\cdot\underline{\nabla}\sigma_t(\underline{r})\right]\psi(\underline{r},\underline{\Omega}) = \sigma_t(\underline{r})Q(\underline{r}). \tag{11}$$

By inserting Eq.(8) into Eq.(11), we get

$$-\underline{\Omega}\cdot\underline{\nabla}\left[\underline{\Omega}\cdot\underline{\nabla}\psi(\underline{r},\underline{\Omega})\right] + \left[\sigma_t^2(\underline{r}) - \underline{\Omega}\cdot\underline{\nabla}\sigma_t(\underline{r})\right]\psi(\underline{r},\underline{\Omega}) = \sigma_t(\underline{r})Q(\underline{r}) - \underline{\Omega}\cdot\underline{\nabla}Q(\underline{r}). \tag{12}$$

Eq.(12) is a modified form for the original SAAF transport equation [Eq.(7)]. Comparing to Eq.(7), the terms with inversed total cross section disappeared from this equation without losing the general quality of the 2nd transport equation. However, an additional term representing the total cross-section derivative [i.e., $\underline{\nabla}\sigma_t(\underline{r})$] appears in the collision term in the modified equation. This term actually will pose some challenges in numerically solving the modified SAAF equation. We will elaborate this in Section 4.

Eq.(7) is really a one-group SAAF formulation that is appropriate for source iteration computation scheme with the S_N method for the angular variable [1]. The operator on the left side of the equation is

self-adjoint and positive-definite though the equation as a whole is not self-adjoint [2]. This property can be extended to the multigroup case straightforwardly. Eq.(12) essentially preserves the self-adjoint feature of Eq.(7) because the operator on the left side of the Eq.(12) has the same characteristics as that of the Eq.(7) if one noticed the derivative operator in the second term only operates on the total cross section. The only disadvantage of the Eq.(7) is that it lacks of conservative property and has only a non-conservative analytic form. This may not considered as a big issue since we mainly use this scheme in reactor physics calculations in which the variation of the solution is well-resolved by the mesh [2].

Without previous awareness, the authors later realized the Modified form of SAAF, Eq.(12), is actually identical to the so-called 'least-square' formulation of the 2nd order neutron transport equation [2, 4]. In this regard, this paper provides an alternative way to derive the least-square form transport equation, in which we use a purely algebraic technique that is more straightforward and simpler than that in Ref. 2.

3. NUMERICAL METHODS TO SOLVE THE MODIFIED SAAF TRANSPORT EQUATION

For demonstration, we consider the modified SAAF equation in a monoenergetic form with one-dimensional slab geometry

$$-\mu^2 \frac{\partial^2}{\partial x^2}\psi(x,\mu)+\left[\sigma_t^2(x)-\mu\frac{d}{dx}\sigma_t(x)\right]\psi(x,\mu)=\sigma_t(x)Q(x)-\mu\frac{d}{dx}Q(x) \tag{13}$$

The discrete ordinate (e.g., S_N) form of the equation is given as

$$-\mu_m^2 \frac{\partial^2}{\partial x^2}\psi_m(x)+\left[\sigma_t^2(x)-\mu_m\frac{d}{dx}\sigma_t(x)\right]\psi_m(x)=\sigma_t(x)Q(x)-\mu_m\frac{d}{dx}Q(x) \tag{14}$$

where the subscribe *m* stands for one polar angle in the neutron transport direction.

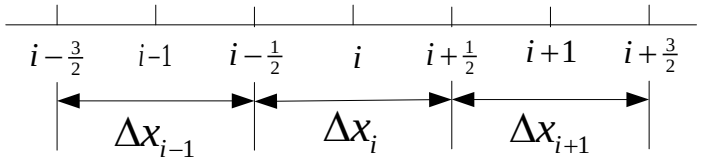

Figure 1. Spatial discretization configuration for the slab geometry.

The spatial discretization configuration of the slab geometry shown in Fig. 1 can be implemented on Eq. (13) in two distinct ways: (a) directly apply it on the Eq.(14); or (b) apply the S_N discretization to the first-order form of the transport equation [Eq.(1)] and then derive a modified SAAF discretization via the same algebraic procedure as shown in Section 2. It needs to mention that the modified SAAF with the second procedure is really derived from the first order form of the transport equation. The first procedure is referred to the direct procedure and the latter as the indirect procedure.

The continuous linear finite element method (CLFEM) scheme for the modified SAAF equation can be obtained via the S_N discretization in a similar way as described in Morel's paper [1]. However, in this work the discretization is performed directly over the Eq. (12) instead of the Eq. (7). For simplicity, the lumped version of the linear-continuous finite element method is first considered for the discretization scheme. Then the indirect procedure is applied on the modified SAAF equation.

3.1 LUMPED CONTINUOUS FINITE ELEMENT METHOD (LCFEM)

With the assumption of piece-wise constant material property, and following Ref. 1, we can obtain the following schemes for the modified SAAF equation:

- The equation for all the cell-edge flux angular fluxes except those on the left ($x = x_{1/2}$) and right ($x = x_{I+1/2}$) boundaries is:

$$A_{i,i-1}\psi_{i-\frac{1}{2}} + A_{i,i}\psi_{m,i+\frac{1}{2}} + A_{i,i+1}\psi_{m,i+\frac{3}{2}} = \sigma_{t,i+\frac{1}{2}} Q_{i+\frac{1}{2}} \Delta x_{i+\frac{1}{2}} - \mu_m \left(Q_{i+1} - Q_i\right) , \tag{15}$$

where

$$A_{i,i-1} = -\frac{\mu_m^2}{\Delta x_i},\ A_{i,i} = \frac{\mu_m^2}{\Delta x_{i+1}} + \frac{\mu_m^2}{\Delta x_i} + \sigma_{t,i+\frac{1}{2}}^2 \Delta x_{i+\frac{1}{2}} - \mu_m \left(\sigma_{t,i+1} - \sigma_{t,i}\right),\ A_{i,i+1} = -\frac{\mu_m^2}{\Delta x_{i+1}}, \tag{16}$$

and

$$\sigma_{t,i+\frac{1}{2}} = \frac{\sigma_{t,i+1}\Delta x_{i+1} + \sigma_{t,i}\Delta x_i}{\Delta x_{i+1} + \Delta x_i},\ Q_{i+\frac{1}{2}} = \frac{Q_{i+1}\Delta x_{i+1} + Q_i \Delta x_i}{\Delta x_{i+1} + \Delta x_i},\ \Delta x_{i+\frac{1}{2}} = \frac{\Delta x_{i+1} + \Delta x_i}{2} . \tag{17}$$

The cross section derivative term $d\sigma_t / dx$ in Eq.(14) is approximated with the first order finite difference scheme, that is

$$\frac{d\sigma_t}{dx} \approx \frac{\sigma_{t,i+1} - \sigma_{t,i}}{\Delta x_{i+\frac{1}{2}}} . \tag{18}$$

- The equation for $\psi_{m,\frac{1}{2}}$ (i.e, the *i=0* node) is

$$A_{0,0}\psi_{m,\frac{1}{2}} + A_{0,1}\psi_{m,\frac{3}{2}} - \mu_m \sigma_{t,1}\psi_{m,L} = \sigma_{t,1} Q_1 \frac{\Delta x_1}{2} - \mu_m Q_1 , \tag{19}$$

where

$$A_{0,0} = \frac{\mu_m^2}{\Delta x_1} + \frac{1}{2}\sigma_{t,1}^2 \Delta x_1,\ A_{0,1} = -\frac{\mu_m^2}{\Delta x_1}, \tag{20}$$

Here, $\psi_{m,L}$ is the angular flux at the left cell-edge with different values for incoming and outgoing directions

$$\psi_{m,L} = \begin{cases} f_m & \mu_m > 0 \\ \psi_{m,\frac{1}{2}} & \mu_m < 0 \end{cases} , \tag{21}$$

where for a source condition f_m denotes the incident flux, for a vacuum condition f_m =0, and for a reflective condition $f_m = \psi_{\frac{1}{2}}(-\mu_m)$.

Two assumptions are considered at the this cell

$$\sigma_{t,\frac{1}{2}} = \sigma_{t,1},\ Q_{\frac{1}{2}} = Q_1 . \tag{22}$$

- The equation for $\psi_{m,I+\frac{1}{2}}$ (i.e., the *i=I* node) is

$$A_{I,I-1}\psi_{m,I-\frac{1}{2}} + A_{I,I}\psi_{m,I+\frac{1}{2}} + \mu_m \sigma_{t,I}\psi_{m,R} = \sigma_{t,I} Q_I \frac{\Delta x_I}{2} + \mu_m Q_I , \tag{23}$$

where

$$A_{I,I-1} = -\frac{\mu_m^2}{\Delta x_I},\ A_{I,I} = \frac{\mu_m^2}{\Delta x_I} + \frac{1}{2}\sigma_{t,I}^2 \Delta x_I , \tag{24}$$

And $\psi_{m,R}$ is defined as follows

$$\psi_{m,R} = \begin{cases} \psi_{m,I+\frac{1}{2}} & \mu_m > 0 \\ f_m, & \mu_m < 0 \end{cases} . \tag{25}$$

Where for a source condition f_m denotes the incident flux, for a vacuum condition $f_m = 0$, and for a reflective condition $f_m = \psi_{I+\frac{1}{2}}(-\mu_m)$. Here we assume at the cell ***I***

$$\sigma_{t,I+\frac{1}{2}} = \sigma_{t,I}, \quad Q_{I+\frac{1}{2}} = Q_I \ . \tag{26}$$

The spatial discretization schemes used in Eq.(15), (19) and (23) are referred to as a lumped linear-continuous finite element method (LCFEM), in which the standard finite element equations are lumped by replacing three-point cell edge removal term with one-point terms. This results in a more robust (i.e., more positive) discretization at the cost of accuracy. Although both the lumped and standard equations are second-order accurate, the error is nonetheless larger for the lumped scheme in the thin-mesh limit [1].

3.2 STANDARD CONTINUOUS FINITE ELEMENT METHOD (SCFEM)

The standard continuous finite element method (SCFEM) discretization can be obtained from the lumped version by making the following substitution: For the removal term in the internal cell,

$$\sigma^2_{t,i+\frac{1}{2}}\psi_{m,i+\frac{1}{2}}\Delta x_{i+\frac{1}{2}} \leftarrow \sigma^2_{t,i}\left(\frac{1}{3}\psi_{m,i-\frac{1}{2}} + \frac{2}{3}\psi_{m,i+\frac{1}{2}}\right)\frac{\Delta x_i}{2} + \sigma^2_{t,i+1}\left(\frac{2}{3}\psi_{m,i+\frac{1}{2}} + \frac{1}{3}\psi_{m,i+\frac{3}{2}}\right)\frac{\Delta x_{i+1}}{2} \ . \tag{27}$$

For the left boundary cell

$$\sigma^2_{t,1}\psi_{m,\frac{1}{2}}\frac{\Delta x_1}{2} \leftarrow \sigma^2_{t,1}\left(\frac{2}{3}\psi_{m,\frac{1}{2}} + \frac{1}{3}\psi_{m,\frac{3}{2}}\right)\frac{\Delta x_1}{2} \ . \tag{28}$$

For the right boundary cell

$$\sigma^2_{t,I}\psi_{m,I+\frac{1}{2}}\frac{\Delta x_I}{2} \leftarrow \sigma^2_{t,I}\left(\frac{1}{3}\psi_{m,I-\frac{1}{2}} + \frac{2}{3}\psi_{m,I+\frac{1}{2}}\right)\frac{\Delta x_I}{2} \ . \tag{29}$$

Consequently, we get the following SCFEM schemes for the modified SAAF equation

- The equation for all the cell-edge angular fluxes in the internal is

$$A_{i,i-1}\psi_{i-\frac{1}{2}} + A_{i,i}\psi_{m,i+\frac{1}{2}} + A_{i,i+1}\psi_{m,i+\frac{3}{2}} = \sigma_{t,i+\frac{1}{2}}Q_{i+\frac{1}{2}}\Delta x_{i+\frac{1}{2}} - \mu_m\left(Q_{i+1} - Q_i\right) \ , \tag{30}$$

where

$$\begin{aligned} A_{i,i-1} &= -\frac{\mu_m^2}{\Delta x_i} + \frac{1}{6}\sigma^2_{t,i}\Delta x_i, \\ A_{i,i} &= \frac{\mu_m^2}{\Delta x_{i+1}} + \frac{\mu_m^2}{\Delta x_i} + \frac{1}{3}\left(\sigma^2_{t,i}\Delta x_i + \sigma^2_{t,i+1}\Delta x_{i+1}\right) - \mu_m\left(\sigma_{t,i+1} - \sigma_{t,i}\right), \\ A_{i,i+1} &= -\frac{\mu_m^2}{\Delta x_{i+1}} + \frac{1}{6}\sigma^2_{t,i+1}\Delta x_{i+1}. \end{aligned} \tag{31}$$

- The equation for $\psi_{m,\frac{1}{2}}$ is

$$A_{0,0}\psi_{m,\frac{1}{2}} + A_{0,1}\psi_{m,\frac{3}{2}} - \mu_m\sigma_{t,1}\psi_{m,L} = \sigma_{t,1}Q_1\frac{\Delta x_1}{2} - \mu_m Q_1 \ , \tag{32}$$

where

$$A_{0,0} = \frac{\mu_m^2}{\Delta x_1} + \frac{1}{3}\sigma^2_{t,1}\Delta x_1, \ A_{0,1} = -\frac{\mu_m^2}{\Delta x_1} + \frac{1}{6}\sigma^2_{t,1}\Delta x_1 . \tag{33}$$

- The equation for $\psi_{m,I+\frac{1}{2}}$ is

$$A_{I,I-1}\psi_{m,I-\frac{1}{2}} + A_{I,I}\psi_{m,I+\frac{1}{2}} + \mu_m\sigma_{t,I}\psi_{m,R} = \sigma_{t,I}Q_I\frac{\Delta x_I}{2} + \mu_m Q_I, \tag{34}$$

where

$$A_{l,l-1} = -\frac{\mu_m^2}{\Delta x_l} + \frac{1}{6}\sigma_{t,l}^2 \Delta x_l, \quad A_{l,l} = \frac{\mu_m^2}{\Delta x_l} + \frac{1}{3}\sigma_{t,l}^2 \Delta x_l . \tag{35}$$

4. PRELIMINARY RESULTS

To assess the viability of the proposed SAAF form and numerical methods, the aforementioned numerical approaches are implemented in a MATLAB code and applied to a couple of one dimensional and one group transport problems described in Reference 2. Both of these problems are fixed source problem with a purposed designed void region, whereas the first problem (Prob.1) includes pure absorber regions and the second problem (Prob. 2) considers the situation where σ_t varies significantly between regions. Both problems consider reflecting B.C. on the left side and vacuum B.C. on the right side. The material properties and domain size of the two problems are shown in Table I and Table II, respectively. The second test problem is really a modified version of the well-known Reed's problem [5]. As can be seen in Table II, significant discontinuities in material properties exist between different regions of the problem.

Table I. Material Properties of the First Problem (Prob. 1).

	Region 1	Region 2	Region 3
S [cm^{-1}s^{-1}]	1	0	0
σ_t [cm^{-1}]	0.5	0	0.8
σ_s [cm^{-1}]	0	0	0
x [cm]	$0 \le x < 2.5$	$2.5 \le x < 7.5$	$7.5 \le x \le 10$

Table II. Material Properties of the Second Problem (Prob. 2).

	Region 1	Region 2	Region 3	Region 4	Region 5
S [cm^{-1}s^{-1}]	100	0	0	0	1
σ_t [cm^{-1}]	100	0	1	5	1
σ_s [cm^{-1}]	0	0	0.9	0	0.9
x [cm]	$0 \le x < 2$	$2 \le x < 4$	$4 \le x < 6$	$6 \le x < 7$	$7 \le x \le 8$

To select a reasonable number of mesh for each region, a quick sensitivity study on the mesh size is performed. The maximum absolute relative error percent for the different mesh size from 10 to 1000 for Prob.1 and from 10 to 2500 for Prob. 2 are calculated and shown in Fig. 2 and 3, respectively. The numerical error used to capture the insensitivity domain in terms of number of mesh is defined as the maximum norm of the relative deviation between the reference solutions and the proposed numerical solutions using the CFEM method. For Prob. 1, we use analytical solutions as the reference solutions. For Prob. 2, the reference solutions are obtained from the difference diamond (DD) scheme on the first order transport equation [6]. All calculations consider S_6 quadrature set for the angular discretization. It is worth to mention that in Prob. 2 the mesh number of Region 1 is fixedly set as 190 and only the mesh number for Region 2 to 5 are changed according to the sensitivity study (see Fig. 3) because we found the results are nearly insensitive the mesh size in Region 1.

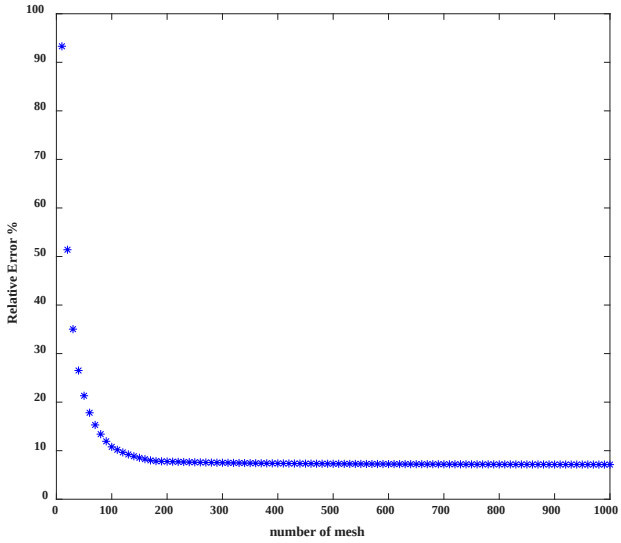

Figure 2. Relative error (%) and different number of mesh for regions 1 to 3 at Prob. 1.

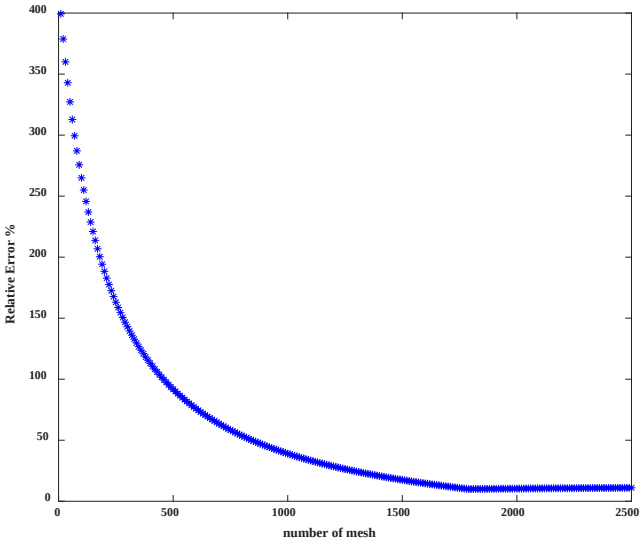

Figure 3. Relative error (%) and different number of mesh for regions 2 to 5 at Prob. 2.

As shown in Fig. 2 and 3, the relative error percent for Prob.1 and Prob. 2 become nearly unchanged after mesh number increased to 300 and 2000, respectively. Therefore these numbers are used as the number of meshes in the numerical calculations.

The analytical solution of scalar flux for the Prob. 1 is given as follow

$$\Phi(x)=\frac{S}{\sigma_{a,1}}\begin{cases}(2-E_2(\sigma_{a,1}(2.5-x))-E_2(\sigma_{a,1}(2.5+x))), & 0\le x<2.5\\ (1-E_2(2\sigma_{a,1}(2.5))), & 2.5\le x<7.5\\ (E_2(\sigma_{a,3}(x-7.5))-E_2(2\sigma_{a,1}(2.5)+\sigma_{a,3}(x-7.5))), & 7.5\le x<10.\end{cases} \tag{36}$$

where E_2 represents the following exponential integral:

$$E_2(x)=\int_1^{\infty}\frac{\exp(-xz)}{z^2}dz \tag{37}$$

The solutions as well as relative error (%) between the analytical method and CFEM for Prob. 1 are given in Fig. 4. All the results provided in this section for the CFEM are merely obtained the standard one (SCFEM) because the lumped scheme gives relatively larger truncation errors due to the thin-mesh limit.

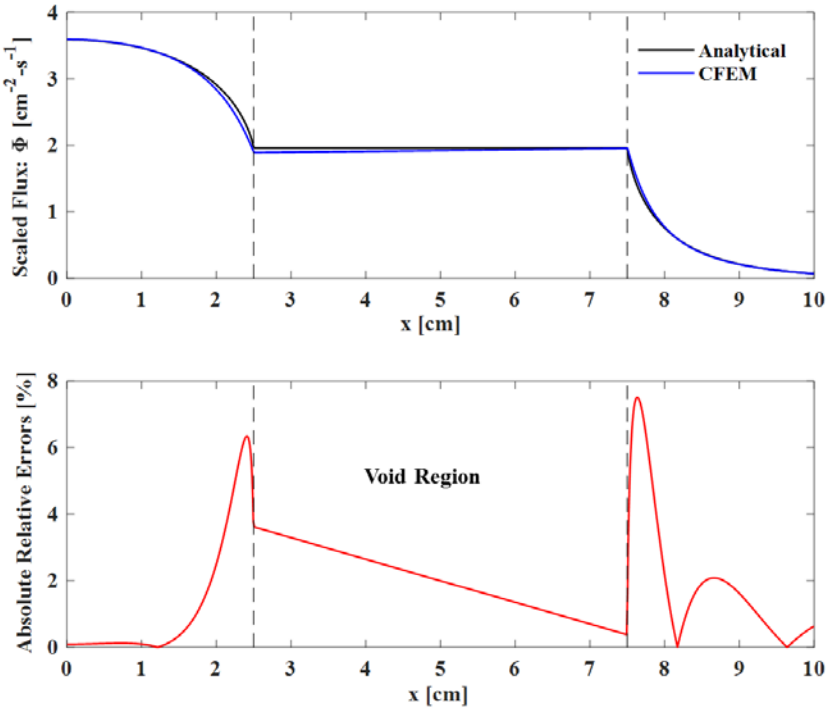

Figure 4. Comparison of the scale flux distribution solution for Prob.1 obtained by the analytical method and the standard CFEM method on modified SAAF equation.

As shown in Fig. 4, with a refined mesh grid, the CFEM solution to the modified form of the SAAF equation reaches the analytical solution with a maximum relative deviation less than 8%, which indicates the modified SAAF can undoubtedly handle problems with regions. It can be seen the large errors normally appears in interfacial areas for this simple 1-D problem, and because of this effect, the solutions adjacent to the interfacial areas usually have larger errors than those far away from the interface regions. The errors in the void region exhibit an interesting linear trend characteristic from one region to another. All the features in the error curve can be possibly attributed to the low accurate approximation of the cross-section derivative term appeared in the modified SAAF equation. Since the material varies in regions, the CFEM cannot accurately catch these variations with a high fidelity modeling of the material change term in the interface areas. As a result, CFEM fails to provide accurate flux solution in these areas. We anticipate the DFEM would be able to demonstrate better results in this regards.

The comparison of the CFEM solution to the Diamond Difference first order transport results for the Prob. 2 is shown Fig. 5.

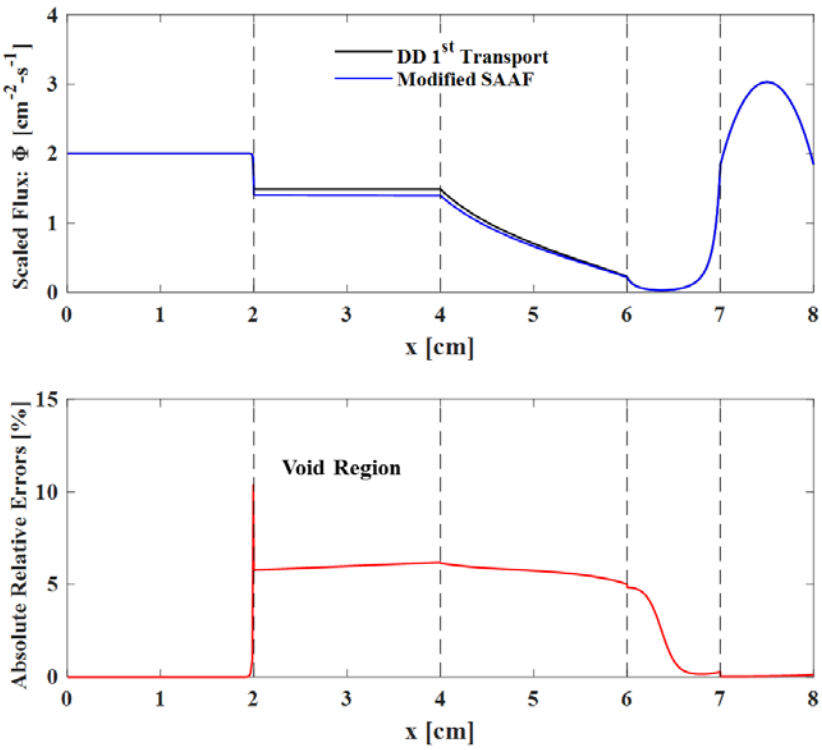

Figure 5. Comparison of the scale flux distribution solution for Prob. 2 obtained by the DD method on standard 1st order transport equation and the standard CFEM method on modified SAAF equation.

The main point of this work is again to provide a modified SAAF equation that demonstrate the capability of nuclear flux calculation at the void region. As the Fig. 5 illustrates, the modified SAAF is providing the neutron flux with the maximum error ~10%. Therefore, the accuracy of results proves the accomplishment of the paper objective, especially at the void region. As shown in Fig. 5, the relative error at the boundary of Region 1 and the void area is varying around 10%., and the errors for the rest of regions are less than 5%.

With the demonstration of these two simple 1-D transport problems, we preliminary conclude the proposed modified SAAF can be an alternative and effective approach to calculate neuron flux for problems with void regions with some extend of accuracy without facing the incompatibility issue arisen with conventional SAAF equation.

5. CONCLUSIONS AND FUTURE EFFORTS

In this paper, we proposed a modified form for the SAAF transport equation to omit the presence of inverse cross section with the objective to make the resulting equation be void compatible. To demonstrate the viability of the proposed equation, two one-dimension one-group fixed source transport problems were solved with standard continues finite element method based on the modified SAAF equation. A quick sensitivity study was performed to provide proper mesh sizes for each test problem. The results of the first test problem, which has a pure absorber region in the middle, show an accuracy above 92%. The outcomes of the second problem, which has different total cross-section at different region, provide a prediction with a high accuracy of 95%. These results indicate the proposed modified SAAF equation in this paper can be used as an alternative 2nd order transport equation in solving flux for problems with void regions.

REFERENCES

1. J. E. Morel and J. M. McGhee, “A Self-Adjoint Angular Flux Equation,” *Nucl. Sci. Eng.*, **132**, 312-325 (1999).
2. J. Hansen, et al, “A Least Squares Transport Equation Compatible with Voids,” *Journal of Computational and Theoretical Transport*, 43, 374-401 (2014).
3. Y. Wang, et al., “Diffusion Acceleration Schemes for Self-Adjoint Angular Flux Formulation with a void Treatment,” *Nucl. Sci. Eng.*, **176**, 201-225 (2014).
4. V. M. Laboure, et al., “Globally Conservative, Hybrid Self-Adjoint Angular Flux and Least-Squares Method Compatible with Voids,” *Nucl. Sci. Eng.*, **185**, 294-306 (2017).
5. W. REED, “New Difference Schemes for the Neutron Transport Equation,” *Nucl. Sci. Eng.*, **46**, 309-314 (1971).
6. E. E. Lewis and J. W. F. Miller, *Computational Methods of Neutron Transport*, Wiley, Hoboken, New Jersey (1984).